

Lecture XII - Metro Passenger Flow Control

Applied Optimization with Julia

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Introduction

FIFA World Cup 2022 in Qatar



Learning Objectives

After this lecture, you will be able to:

- **Model** time-expanded passenger flows with precomputed delays
- **Translate** a safety goal into inflow control constraints
- **Explain** how the set $\mathcal{R}_{e,t}$ connects periods and minutes
- **Embed** an optimization model into a rolling-horizon heuristic

Public transport

- FIFA World Cup 2022 took place in a **small region**
- The capacity of the metro system was **finite**
- More than **1 million tourists** were expected
- Metro usage was **free for all ticket holders**
- Transport methods were expected to be **overloaded**

Question: What could become an issue?

Crowd Disasters

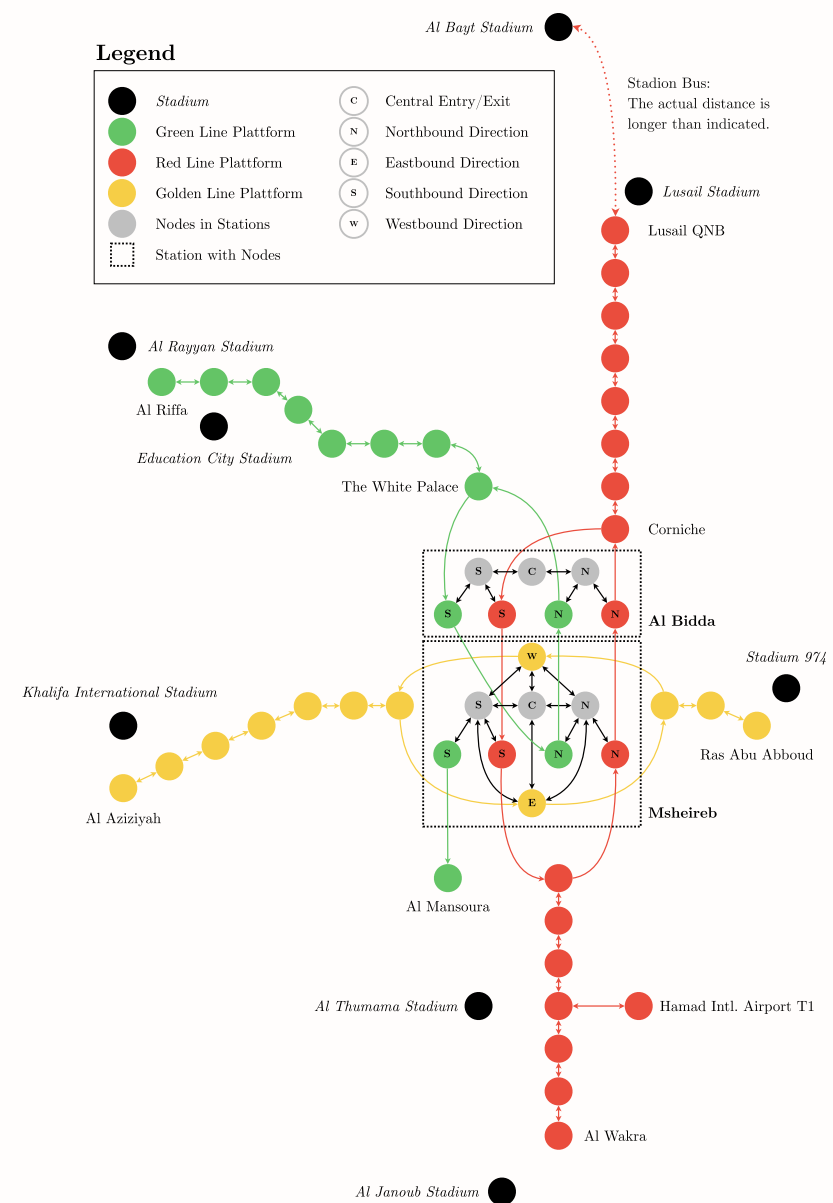


Potential locations

Question: Where could crowd disasters happen?

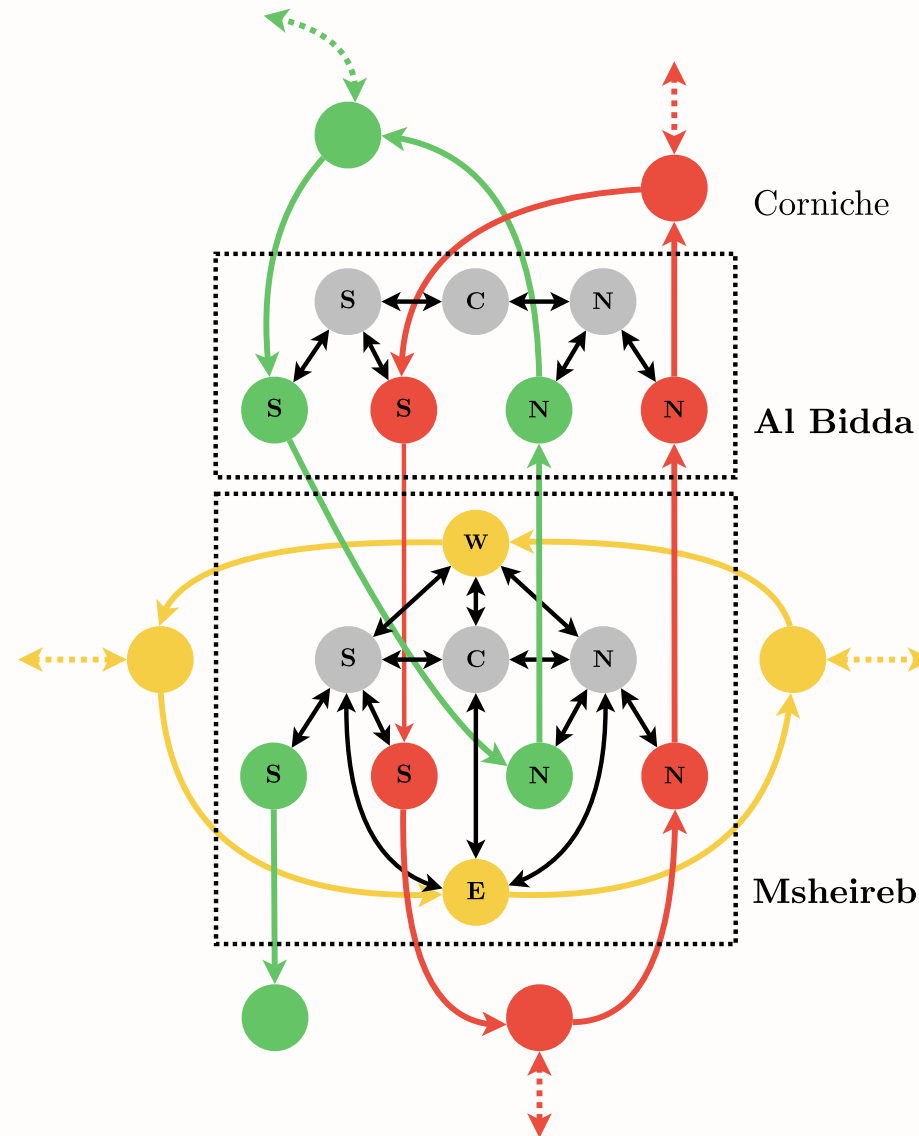
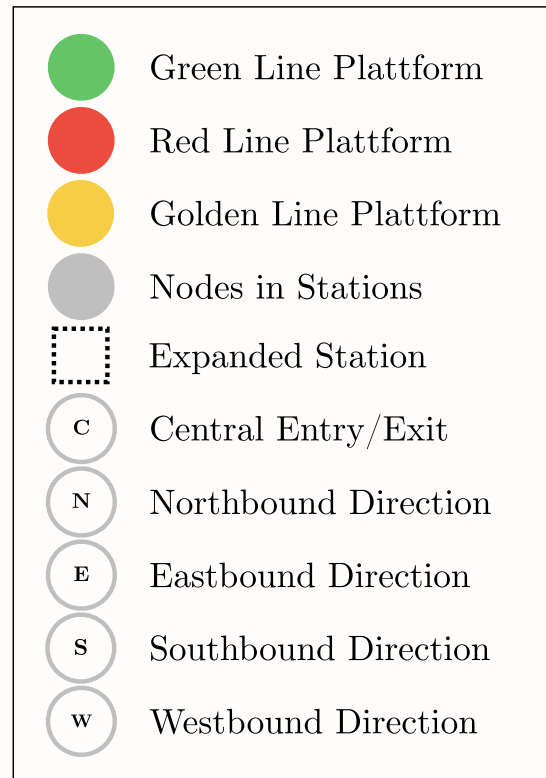
- At event venues and in waiting areas
- At intersections of multiple pedestrian flows
- In narrow passages and entrances
- On crowded metro platforms and transfer stations
- At ticket/turnstile bottlenecks and emergency exits

Metrosystem of Doha



Closer look at the Metro System

Legend



Main Bottleneck

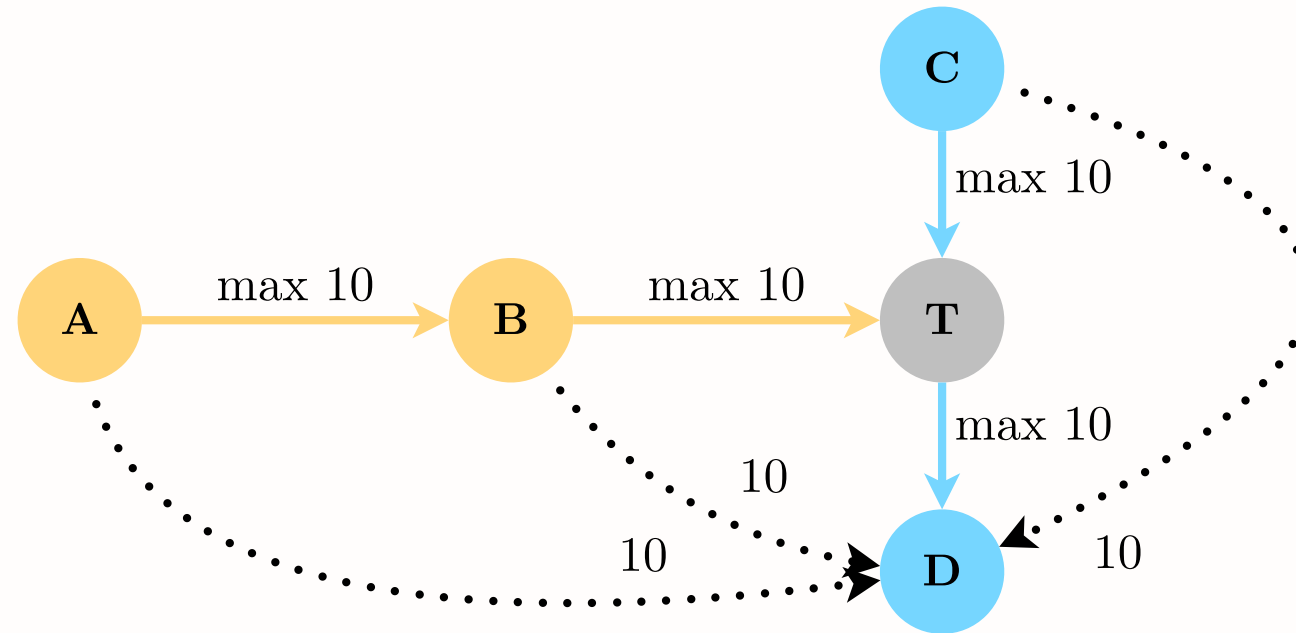
Question: What could be a potential bottleneck?

- Red Line has twice the capacity of Green and Gold Line
- People use the metro to get to the event venues

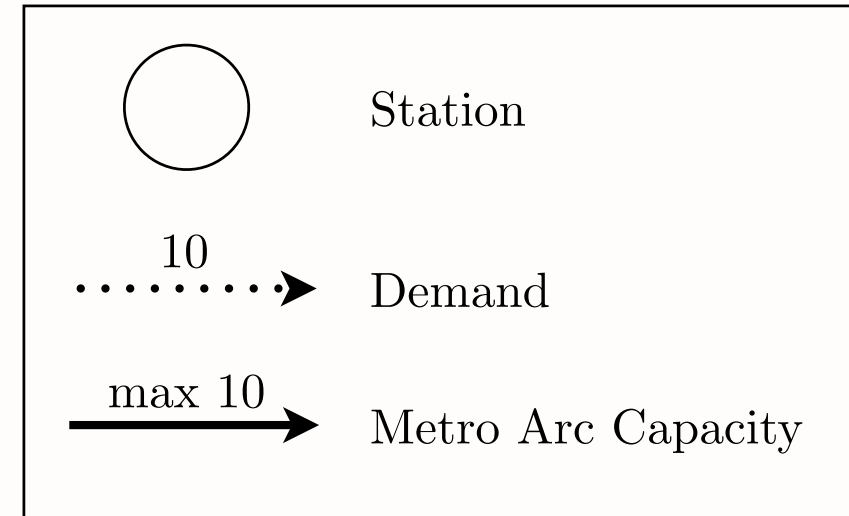
Question: What could be the issue?

- Imagine a match at a stadium along the Gold line
- People from Red and Green line will try to get there
- This will cause a **massive crowd at transfer stations**

Visualization



Legend



- 10 people want to start per minute at **each station**
- Everybody wants to get to **station D**

Capacities inside the stations

- Capacities **inside the stations** are limited as well
- These include the escalators, stairs, and elevators
- But also the platforms and the ticket gates



This can lead to **overcrowding and potential crowd disasters!**

General Issues

Question: What could also be an issue?

- Often unknown **how many people** exactly will participate
- Gathering data is **extremely important and difficult**
- Crowd behavior can be **unpredictable and dynamic**
- Weather conditions may affect **transportation preferences**
- Cultural factors influence **crowd movement patterns**
- Emergency situations require **flexible contingency plans**

Let's take a
closer look at the
problem structure!

Problem Structure

Objective

Question: What could be the objectives of the authorities?

- A **safe and successful event** as host country
- Good **publicity and a positive recognition** worldwide
- **Satisfied visitors** that enjoyed their time

 Note: We can model none of the above directly!

But we can assist in the estimation of transport demands and the design of operational plans to ensure public **safety** and a **smooth transportation** through the city.

Underlying Problem

Question: Can we just start modeling?

- First, we need to understand the **movement patterns**
- Event is **unprecedented**, movement patterns are unknown
- Multiple **concurrent events** affect flow patterns



Tip: We need to estimate the data first!

This is a **huge challenge**, but we can use **simulation** to estimate the data.

Simulation

- Based on **publicly available data** from the area
- Simulates **all individuals** participating at the event
- Includes **all transportation infrastructure**
- **Individual mode choice** based on a choice model



Tip: Julia is a great tool for this!

The simulation was built in Julia. With 1,000,000 individuals walking, using cars, buses, and the metro, simulating a day took **less than 5 minutes**.

Results of the Simulation



- Detailed movement patterns throughout the region
- Potential sections at risk in the transport infrastructure
- Potential capacity overloads at event locations

But it is still
based on a lot
of assumptions!

Main risk: Metro



Idea

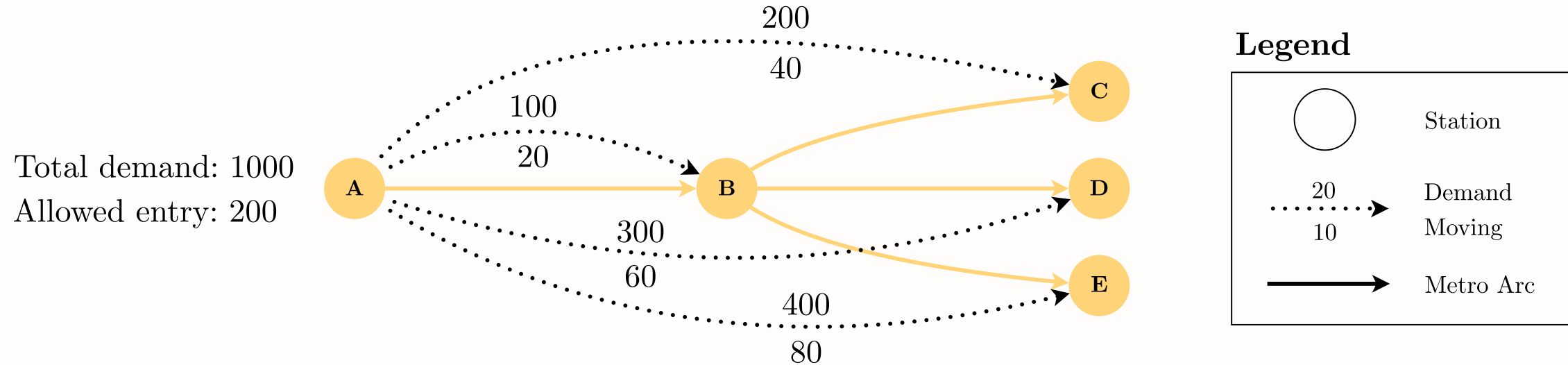
Question: What can we do to prevent crowd disasters?

- Regulate the **inflow** at each individual station
- Ensure utilized capacity is **always within bounds**

Question: What could we try to model?

- **Minimize the queues** outside of the metro stations
- Based on the **allowed inflow** and origin-destination data
- Adhering to the **capacity constraints**

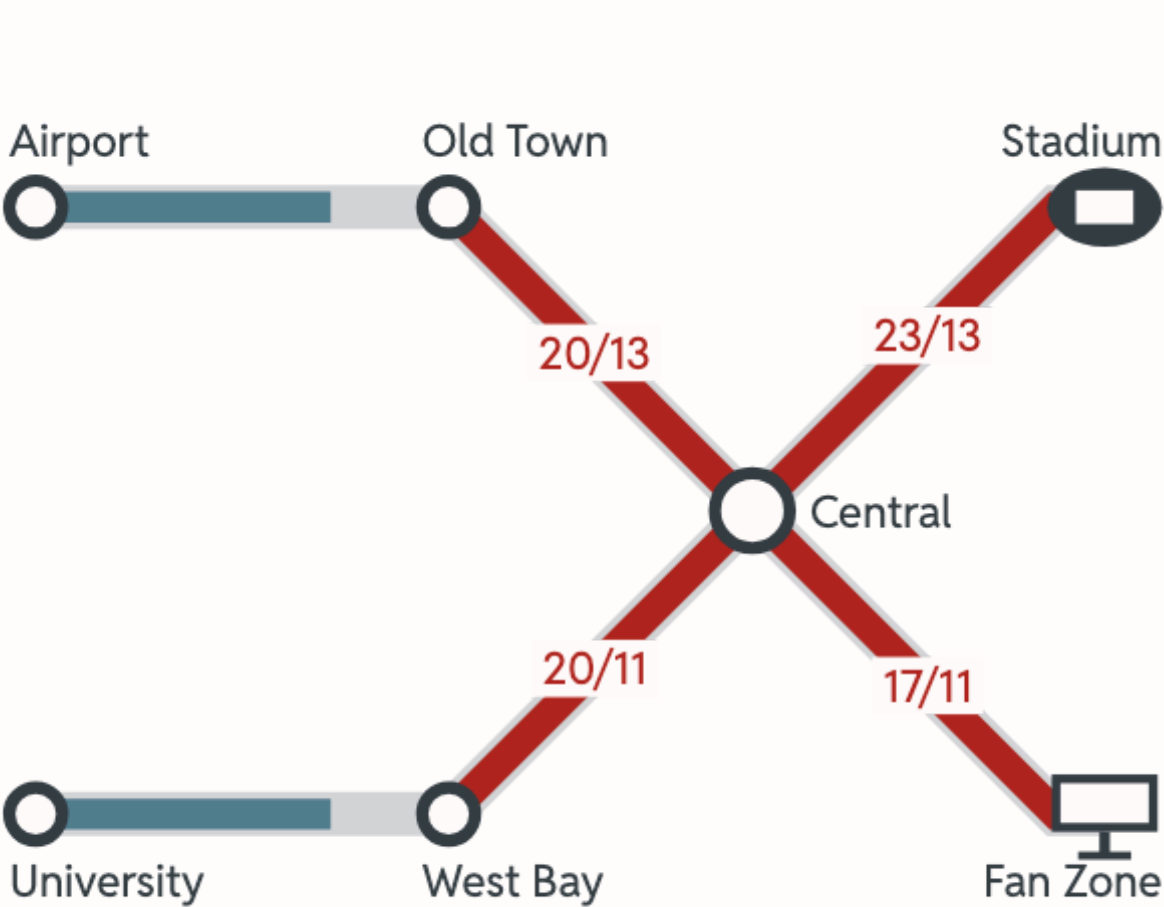
Difficulty



- Patterns have different origin-destination pairs
- Regulating the inflow does not affect the destination

Your Turn

Match day: fans queue at four metro stations. Set each station's gate, the riders it admits per minute; they spread over the stadium and the fan zone by the station's mix and load every track on their way. Admit as many as the tracks take.



Airport	10/min
90% to the stadium	
Old Town	10/min
40% to the stadium	
University	10/min
30% to the stadium	
West Bay	10/min
70% to the stadium	
4 tracks are over capacity.	



You
40 riders per minute

Old Town to Central:
20 riders per minute,
but it takes 13

Optimize

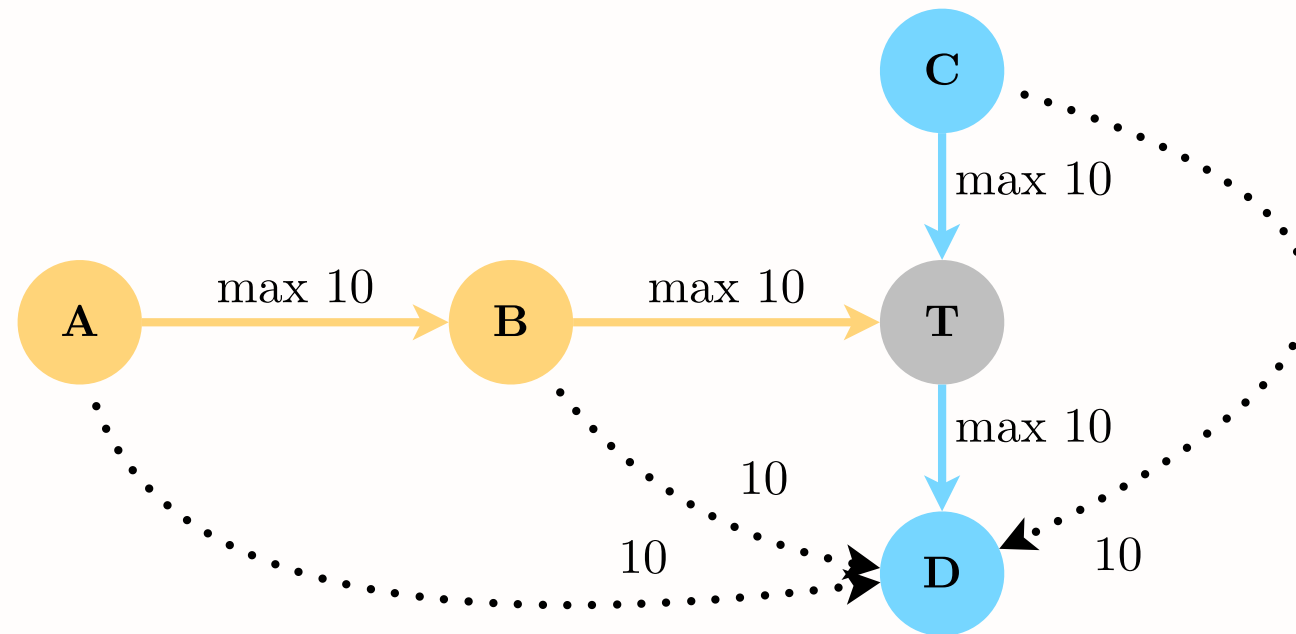
Reset

New

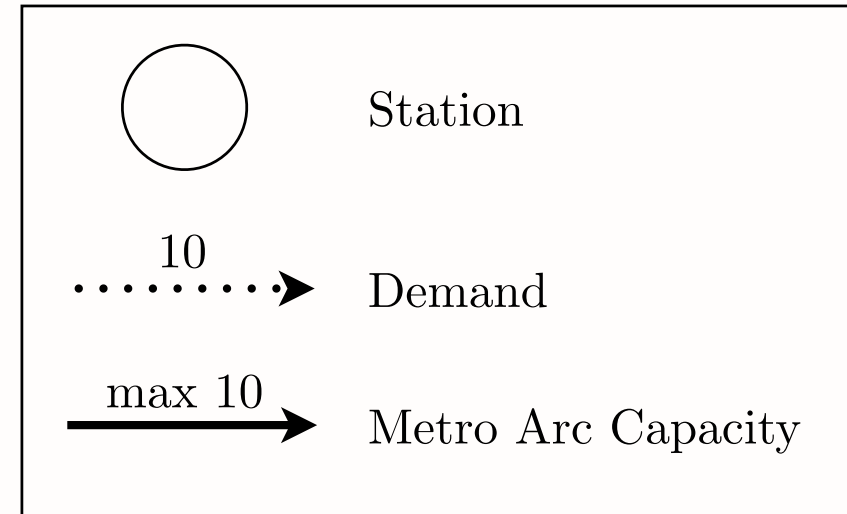
Model Formulation

Graph Sets?

Question: What could be the sets for the graph?



Legend



Graph Sets?

- $\mathcal{G}$ - Connected digraph of the metro network $\mathcal{G} = (\mathcal{O}, \mathcal{E})$
- $\mathcal{O}$ - Set of metro stations, indexed by o
- $\mathcal{E}$ - Set of directed arcs between connected stations

 **Note:** These are the easier sets.

These simply help us to represent the entire metro system as a graph with different nodes $\mathcal{O}$ and arcs $\mathcal{E}$ between connected stations.

Time Sets?

Question: What could be the time sets?

- $\mathcal{T}$ - Set of minutes in the time horizon, indexed by t
- $\mathcal{P}$ - Set of periods in the observed time horizon, where $p \in \{1, 2, \dots, n\}$

 Note: Number of periods

We further define n as the number of periods in observed time horizon.

Periods

Question: Why do we add periods here?

- Staff needs **clear**, consistent instructions
- Frequent changes in flow **increase risk of errors**
- **Easier** to manage and communicate for planners

Note: Period Length

We define the period length as the length of the period in minutes measured as m minutes, which is the **same for all periods**.

Mapping Minutes to Periods

- We can define an **additional set** with their relation
- It specifies the relation of **periods p to minutes t** .

$$I_p = \{t \in \mathcal{T} \mid (p - 1) \times m + 1 \leq t \leq p \times m\} \quad \forall p \in \mathcal{P}$$

Question: Can anybody explain it?

- I_p is the set of minutes t that belong to period p
- Minutes are **not overlapping**, but they are **continuous**

Parameters?

Question: What could be possible parameters?

- $q_{o,d,p}$ - Queued passengers from station o to d with $o, d \in \mathcal{O}$ in p
- τ_e - Travel time (min) of the arcs $e \in \mathcal{E}$
- c_e - Max. allowed arc entry rate e per minute with $e \in \mathcal{E}$
- c_o^{min} - Min. station entry rate o per minute with $o \in \mathcal{O}$
- c_o^{max} - Max. station entry rate o per minute with $o \in \mathcal{O}$
- α - Safety factor applied to all capacities ($0 < \alpha < 1$)

Metro Movement

Question: How do people move inside the metro?

- Simple network: assume people use the **shortest path**
- Use **Dijkstra's algorithm** to compute it
- From each station to all other stations



Tip: We can save the results in a new set $\mathcal{C}_{o,d}$

This will help us to model the movement inside the metro.

Shortest Paths (SP)

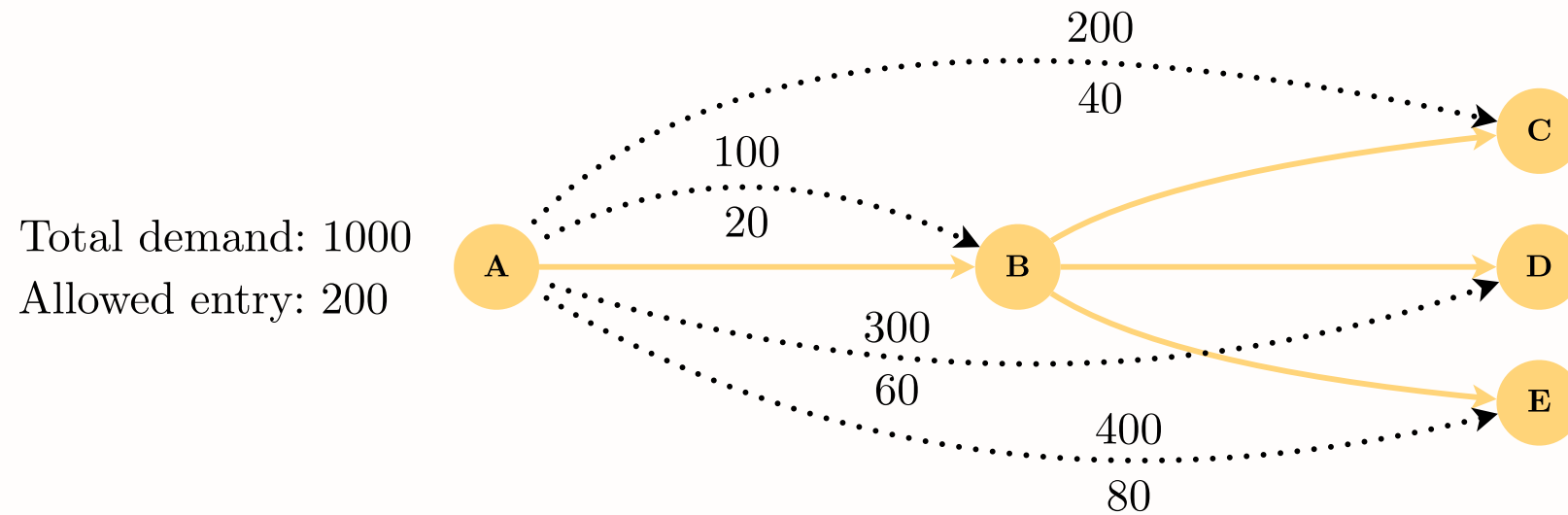
- $\mathcal{C}_{o,d}$ - Set of arcs $e \in \mathcal{E}$ on the SP from o to d with $o, d \in \mathcal{O}$

i The arc travel times τ_e are only the **input** to Dijkstra's algorithm. From the resulting paths we compute the travel time parameter the model actually uses:

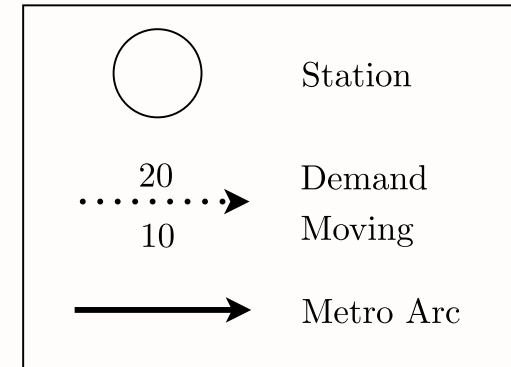
- $\tau_{o,e}$ - Travel time (min) on the SP from station $o \in \mathcal{O}$ to arc $e \in \mathcal{E}$

People Spreading

Question: How do people spread?



Legend



Ratio of Origin-Destination

- We **cannot control** the destination of the passengers
- Thus we assume that people spread **based on destination**

$$\frac{q_{o,d,p}}{\sum_{f \in \mathcal{O}} q_{o,f,p}} \quad \forall o, d \in \mathcal{O}, p \in \mathcal{P}$$

i Based on the ratio of the different destinations d to the total queue for each station $o \in \mathcal{O}$ in each period $p \in \mathcal{P}$. Note that the summation needs its **own index** f over all destinations, as d is already taken by the ratio's destination.

Variables & Objective

Decision Variable?

 Our goal is to:

Regulate the **inflow** at **each individual station** to **minimize the queues** outside the metro stations based on the allowed inflow while **adhering to the capacity constraints**.

 There is **one queue for all passenger flow directions**, as managing multiple queues would be too complex for the planners!

Decision Variable

i We need the following sets:

- All the metro stations, $o \in \mathcal{O}$
- All periods under observation, $p \in \mathcal{P}$

Question: What could be our decision variable?

- $X_{o,p}$ - Allowed inflow (per minute) at station o in period p

Objective Function?

! Our main objective is to:

Regulate the **inflow** at **each individual station** to **minimize the queues** outside the metro stations based on the allowed inflow while **adhering to the capacity constraints**.

Question: How again are queues minimized?

- By the allowed inflow $X_{o,p}$ subtracted from the queue

Objective Function

i We need the following parameters and variables:

- $q_{o,d,p}$ - People queued to travel from station o to d with $o, d \in \mathcal{O}$ in period p
- $X_{o,p}$ - Allowed inflow (per minute) at station o in period p

Question: What could be our objective function?

$$\text{Minimize} \quad \sum_{o \in \mathcal{O}} \sum_{p \in \mathcal{P}} \left(\sum_{d \in \mathcal{O}} q_{o,d,p} - m \times X_{o,p} \right)$$

Objective Function Revisited

Question: What does the solver really maximize here?

- The queued passengers $q_{o,d,p}$ are **parameters**, not variables
- Their sum is a **constant** — the solver cannot change it

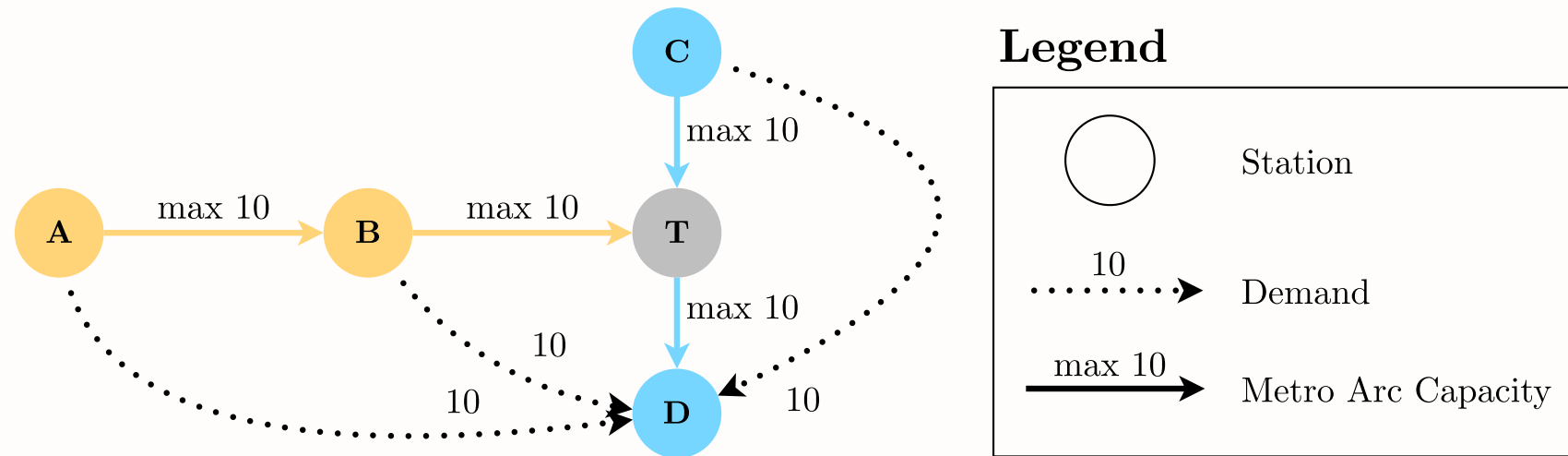
i Note: Minimizing queues = maximizing admissions

Minimizing the objective is exactly the same as **maximizing the total admitted passengers** $\sum_{o \in \mathcal{O}} \sum_{p \in \mathcal{P}} m \times X_{o,p}$. This is also why the model gets away without explicit queue dynamics.

Constraints

Necessary Constraints

Question: What constraints do we need?



- The capacity of each arc is **not exceeded**
- Do not dispatch more people than are queued
- Do not dispatch less than the minimum allowed inflow

Things are going to
get a little
complicated now!

Central Question

Question: When do people flowing into the metro system change the arcs?

- People enter metro station at o with destination d
- They will lead to a usage of an arc e on their SP
- This usage depends on their path and the travel times

 **Tip:** You'll likely know what we need now!

We can add a new set $\mathcal{R}_{e,t}$ to help us with this.

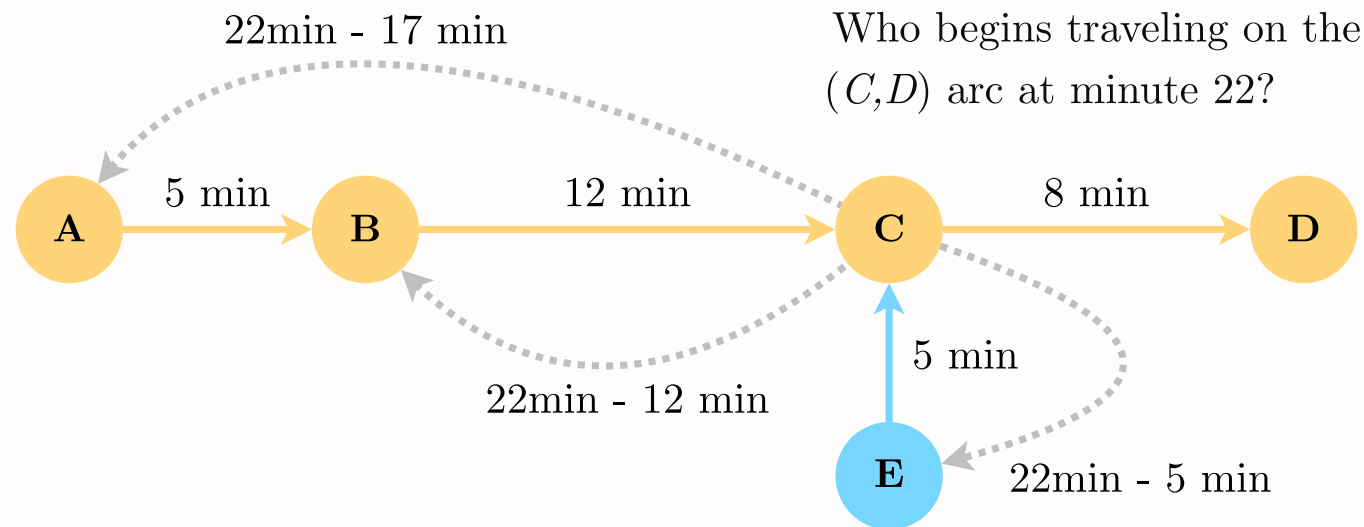
Set of Time-Delays

$$\mathcal{R}_{e,t} = \{(o, d, p) \mid o, d \in \mathcal{O}, p \in \mathcal{P}, q_{o,d,p} > 0, \\ e \in \mathcal{C}_{o,d}, t - \tau_{o,e} \in I_p\} \quad \forall e \in \mathcal{E}, t \in \mathcal{T}$$

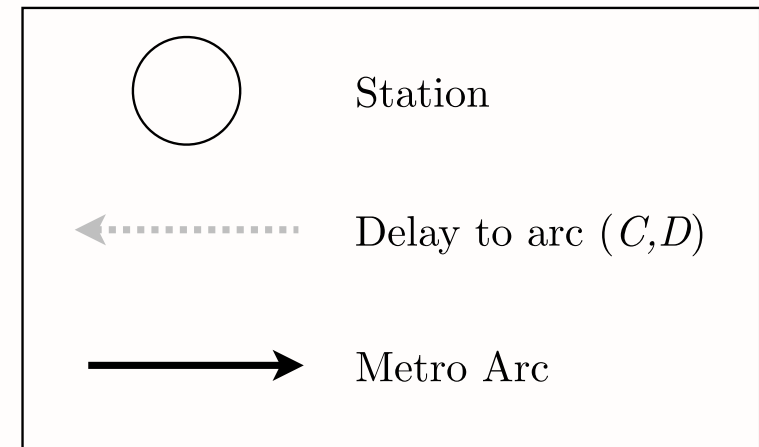
Question: Who can explain this set?

The set $\mathcal{R}_{e,t}$ contains **all combinations** (o, d, p) which trigger a **capacity utilization** of arc e in minute t .

Small Example



Legend



The set contains all o-d pairs and periods that result in passengers entering arc (C, D) at minute 22. With $m = 2$, it is $\{(A, D, 3), (B, D, 5), (C, D, 11), (E, D, 9)\}$.

Check it yourself, e.g. from A : $\tau_{A,(C,D)} = 5 + 12 = 17$ and $22 - 17 = 5 \in I_3 = \{5, 6\}$.

Essentially, we can use this set to compute the utilization of an arc at each minute based on the inflow level at the different stations and periods.

Ensure Capacity Utilization?

! The goal of this constraint is to:

Ensure that the capacity of each arc is not exceeded at any minute.

i We need the following variable, parameter and set:

- $q_{o,d,p}$ - people queued to travel from station $o \in \mathcal{O}$ to station $d \in \mathcal{O}$ in p
- c_e - people max. allowed to enter arc e per minute with $e \in \mathcal{E}$
- $\mathcal{R}_{e,t}$ - mapping of station entries to arc e in minute t with $e \in \mathcal{E}$ and $t \in \mathcal{T}$
- α - safety factor applied to all capacities ($0 < \alpha < 1$)
- $X_{o,p}$ - the allowed inflow per minute at metro station o in the period p

Ensure Capacity Utilization

Question: What could be the constraint?

$$\sum_{(o,d,p) \in \mathcal{R}_{e,t}} X_{o,p} \times \frac{q_{o,d,p}}{\sum_{f \in \mathcal{O}} q_{o,f,p}} \leq \alpha \times c_e \quad \forall e \in \mathcal{E}, t \in \mathcal{T}$$

i Here, we combine the inflow at each station based on the o-d proportion and let the people spread through the network, checking that no arc is over-utilized at any minute.

Bound the Inflow Rate?

! The goal of this constraint is to:

Ensure that the inflow rate at each station stays within operational bounds.

i We need the following variables and parameters:

- $X_{o,p}$ - Allowed inflow (per minute) at station o in period p
- c_o^{min} - Min. station entry rate o per minute
- c_o^{max} - Max. station entry rate o per minute
- α - Safety factor applied to all capacities ($0 < \alpha < 1$)

Bound the Inflow Rate

Question: What could be the constraint?

$$c_o^{min} \leq X_{o,p} \leq \alpha \times c_o^{max} \quad \forall o \in \mathcal{O}, p \in \mathcal{P}$$

- i* This constraint bounds the inflow rate at each station between the **minimum** and **maximum** allowed entry rates, ensuring compliance with **operational capacity limits**.

Dispatch Only Available People?

! The goal of this constraint is to:

Ensure that we do not dispatch more people than are queued.

i We need the following variables and parameters:

- $X_{o,p}$ - Allowed inflow (per minute) at station o in period p
- $q_{o,d,p}$ - People queued to travel from station o to d with $o, d \in \mathcal{O}$ in p
- m - period length in minutes

Dispatch Only Available People

Question: What could be the constraint?

$$\sum_{d \in \mathcal{O}} q_{o,d,p} - m \times X_{o,p} \geq 0 \quad \forall o \in \mathcal{O}, p \in \mathcal{P}$$

i This constraint ensures **non-negative remaining queues**, preventing the solver from dispatching more passengers than available in the queue.

A Hidden Pitfall

Question: What if a queue is smaller than $m \times c_o^{min}$?

- The lower bound forces $m \times X_{o,p} \geq m \times c_o^{min}$
- But we may only dispatch up to $\sum_{d \in \mathcal{O}} q_{o,d,p}$ people
- Example: $c_o^{min} = 2$, $m = 15$, but only 20 people queued
- A quiet station late at night makes the model **infeasible**!

Question: How could we fix this?

- Set $c_o^{min} = 0$, cap the bound at the queue, or soften the constraint

Metro Inflow Model

$$\text{Minimize } \sum_{o \in \mathcal{O}} \sum_{p \in \mathcal{P}} \left(\sum_{d \in \mathcal{O}} q_{o,d,p} - m \times X_{o,p} \right)$$

subject to:

$$\begin{aligned} \sum_{(o,d,p) \in \mathcal{R}_{e,t}} X_{o,p} \times \frac{q_{o,d,p}}{\sum_{f \in \mathcal{O}} q_{o,f,p}} &\leq \alpha \times c_e && \forall e \in \mathcal{E}, t \in \mathcal{T} \\ c_o^{\min} \leq X_{o,p} &\leq \alpha \times c_o^{\max} && \forall o \in \mathcal{O}, p \in \mathcal{P} \\ \sum_{d \in \mathcal{O}} q_{o,d,p} - m \times X_{o,p} &\geq 0 && \forall o \in \mathcal{O}, p \in \mathcal{P} \\ X_{o,p} &\geq 0 && \forall o \in \mathcal{O}, p \in \mathcal{P} \end{aligned}$$

Model Characteristics

Characteristics

Questions: On model characteristics

- Is the model formulation linear/ non-linear?
- What kind of variable domains do we have?

Note: The answers

It is a **linear program**, as objective and constraints are linear. The variable $X_{o,p} \geq 0$ is **continuous** — fractional inflow rates per minute are perfectly acceptable here.

Model Assumptions

Questions: On model assumptions

- What assumptions have we made?
- What are likely issues that can arise if applied?
- Have we thought in detail about queues?
- Are shortest paths a feasible assumption?

In Practice

Metro Inflow Optimization

Question: Can this be applied?



Metro Inflow Problem

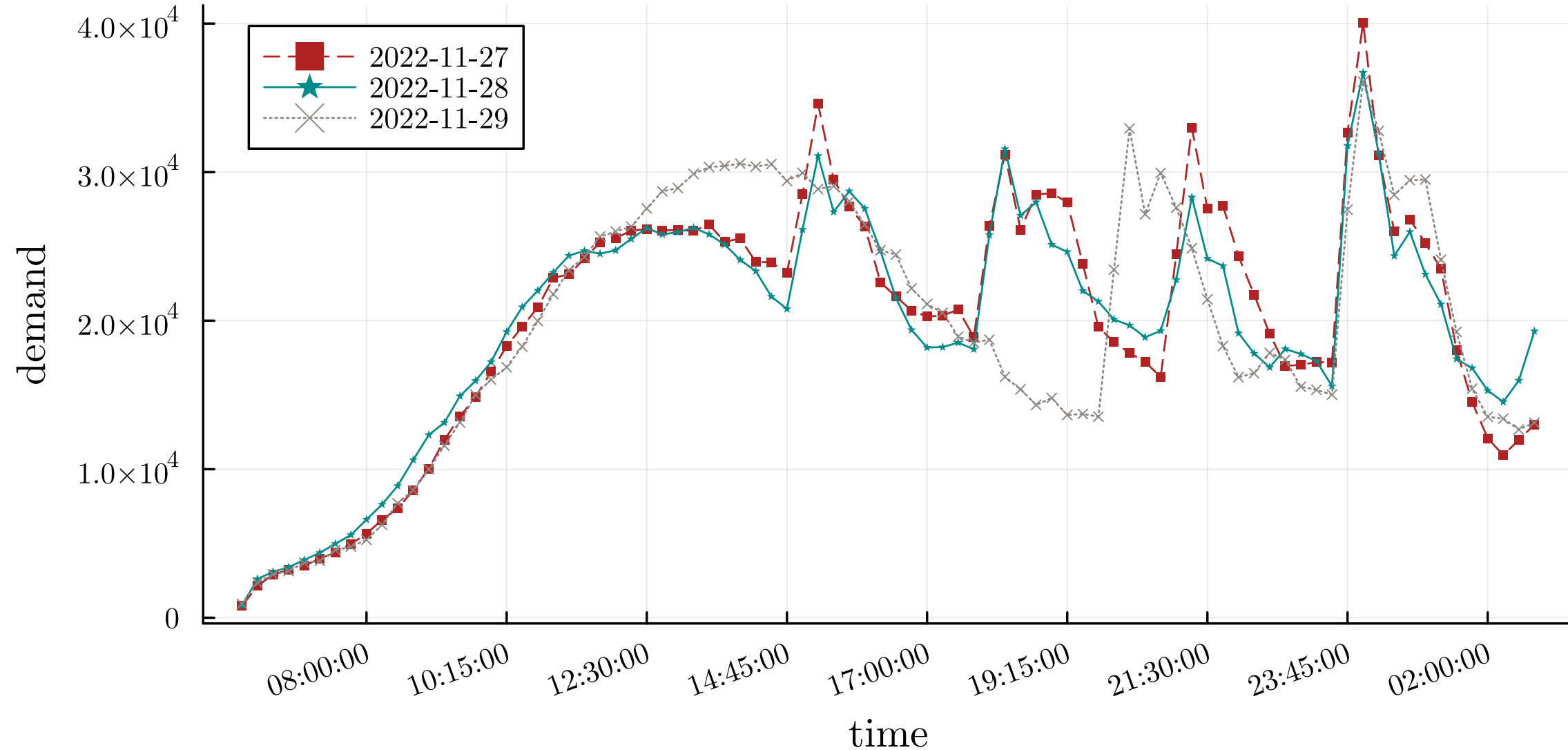
- Solved very fast **within seconds** for realistic problem sizes
- **But** we cannot plan or control the metro inflow
- Queues are **too simplified** with passengers disappearing

Question: Any ideas, how the current model could be improved or how it could be embedded into a heuristic?

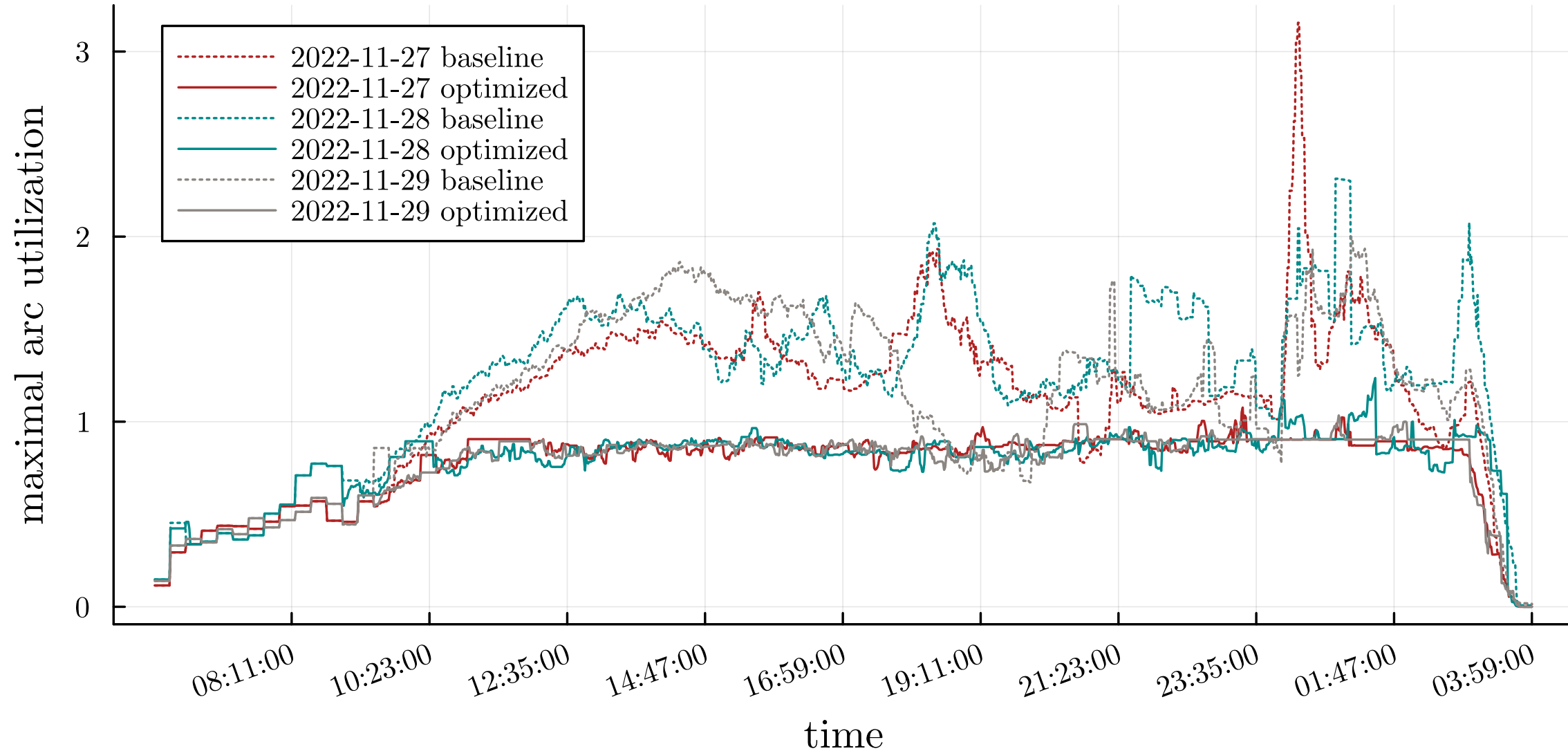
Heuristic: Step-Wise

- Solve the model for the time-horizon of a **few periods**
- Fix the inflow in the **current first period**
- Decrease capacity in the network **based on the inflow**
- Transfer **remaining queues** into the subsequent period
- **Solve the model again**
- **Repeat**, until the inflow is computed for all periods

Transport Demand



Utilization Analysis



Key Results

- Model reduces capacity violations from **25,941 to just 4**
- Maximum arc utilization drops from **316% to 109%**
- **81% of passengers** accommodated safely
- Computation time: **~1.4 seconds** per planning period

Note: Safety vs. Throughput Tradeoff

The remaining 19% of passengers experience external queuing - this is the price for preventing dangerous overcrowding inside the system.

Didn't We Bound the Utilization?

Question: Our constraint enforces $\alpha \times c_e$ with $\alpha < 1$. How can utilization still reach 109%?

- The constraint binds the model for the **forecast** demand
- The 109% occurs in the **rolling-horizon simulation** with realized demand
- Realized flows can deviate from the forecast — but far less than **316%**

An Ordinary Day in Shanghai

Question: Does inflow control also work without a World Cup?



Photo: [cal_gao](#), Unsplash

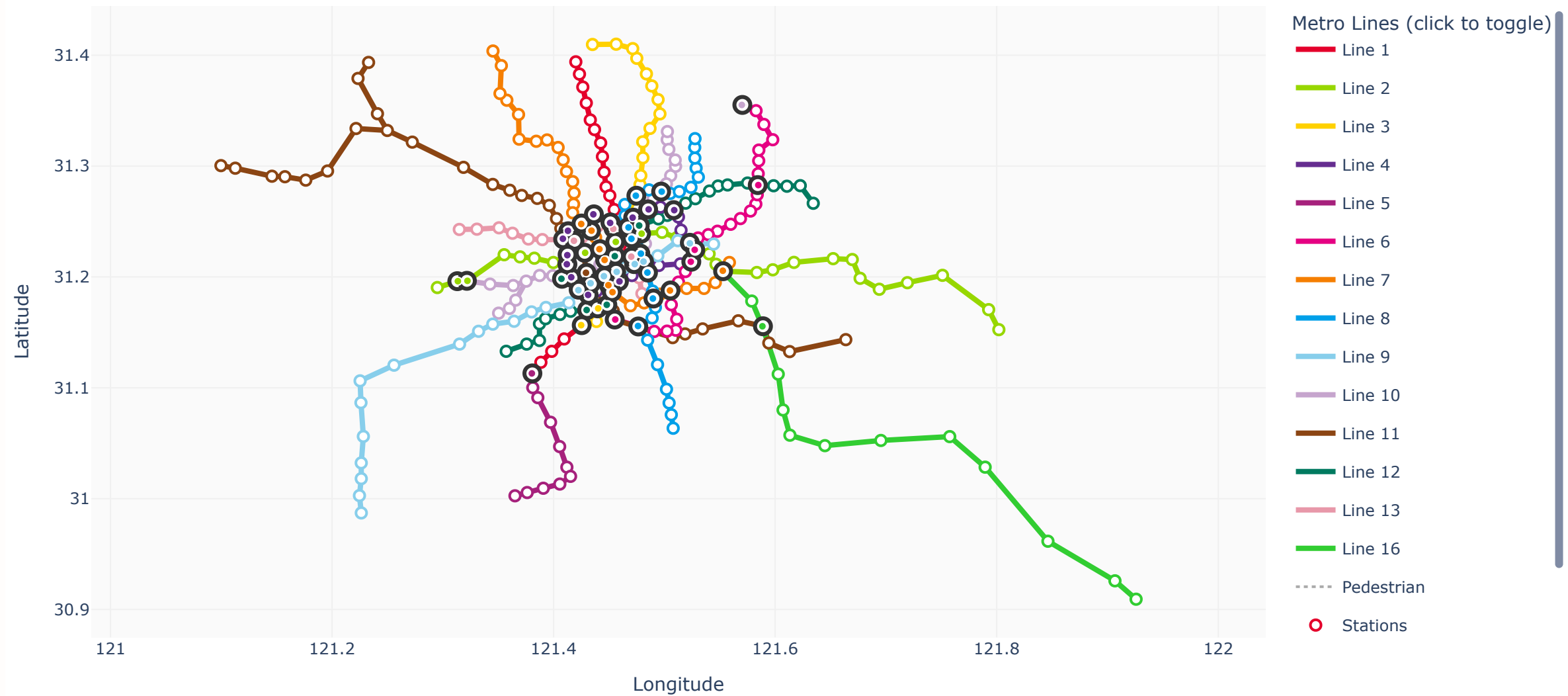
Scalability: Shanghai Metro

- Validated on **302 stations** (~8× more than Doha's 37)
- **99% reduction** in capacity violations
- Computation: **< 2 minutes** per period
- Works for routine **peak-hour demand**

Tip: Generalizable Approach

The method scales from event-driven demand (Qatar World Cup) to routine metropolitan operations (Shanghai rush hour).

Shanghai Metro Network (Expanded Model) - Hover for Details



[Open the Shanghai network visualization in a new tab](#)

Implementation

- Assumption of known destinations is strong
- Movements seemed to follow our forecasts
- We did achieve our goal of metro inflow control
- Simulation was used to estimate the inflows

 Few dangerous situations, especially at the FIFA Fan Fest, were handled well by the authorities.

Comparison with Others

Feature	Other Methods	Our Approach
FIFO Enforcement	Rarely	Yes
Single Queue	Often separate	Yes
Real-Time Capable	Some	Yes (seconds to minutes)
Network Scale	Up to 327 stations	Comparable (302 stations)

Note: State-of-the-Art

First network-level approach combining FIFO enforcement, single-queue modeling, and real-time computational performance.

Wrap Up

- Model can help to achieve a **good balance**
- Can be adapted easily to **any metro system worldwide**
- Especially interesting for **larger Asian cities**

 And that's it for today's lecture!

We now have covered a metro inflow control problem based on a real-world application and are ready to start solving some new tasks in the upcoming tutorial.

Questions?

Literature

Literature I

For more interesting literature to learn more about Julia, take a look at the [literature list](#) of this course.