

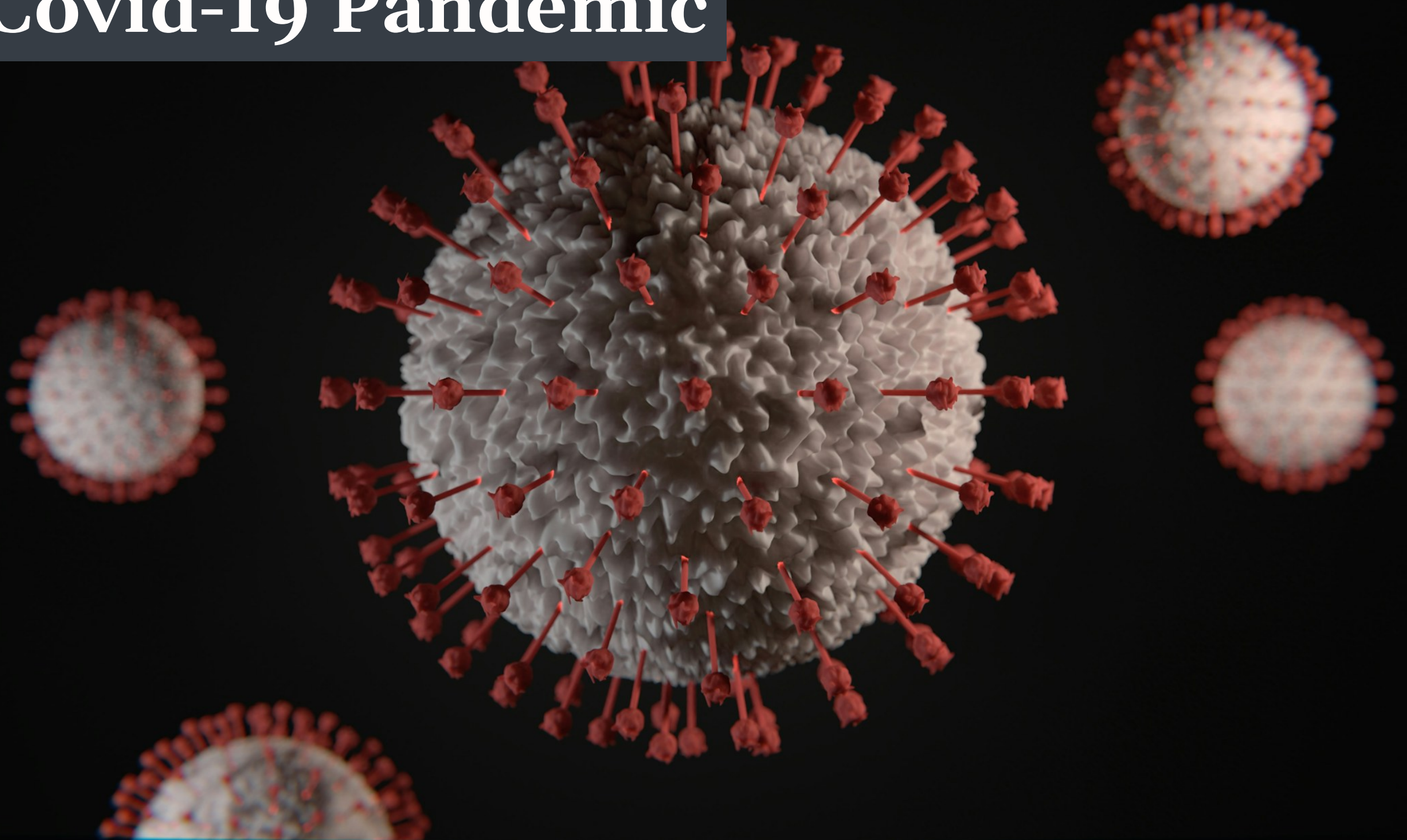
Lecture XI - Arena Seat Planning under Distancing Rules

Applied Optimization with Julia

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Introduction

Covid-19 Pandemic



Challenges for Live Events

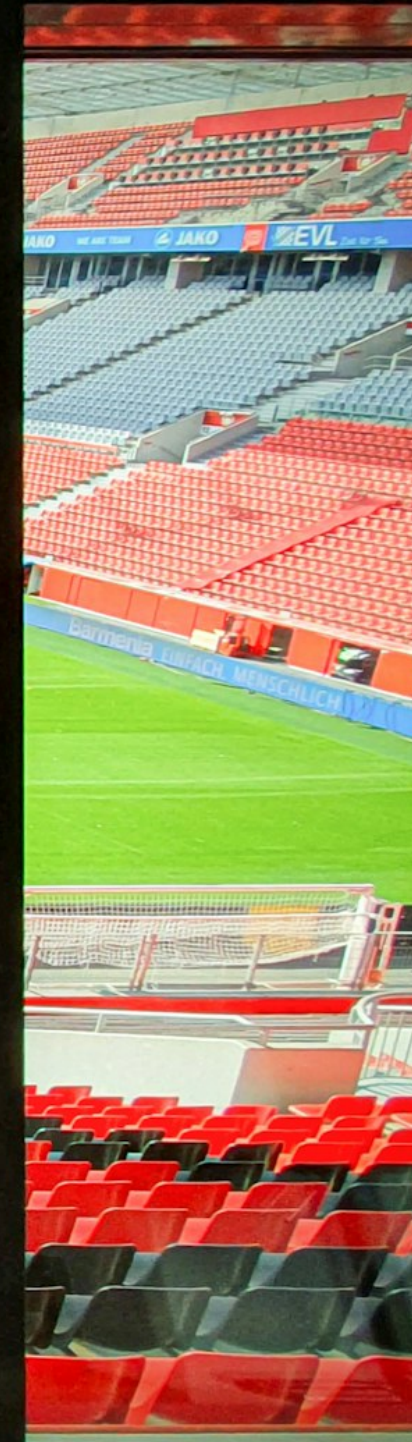
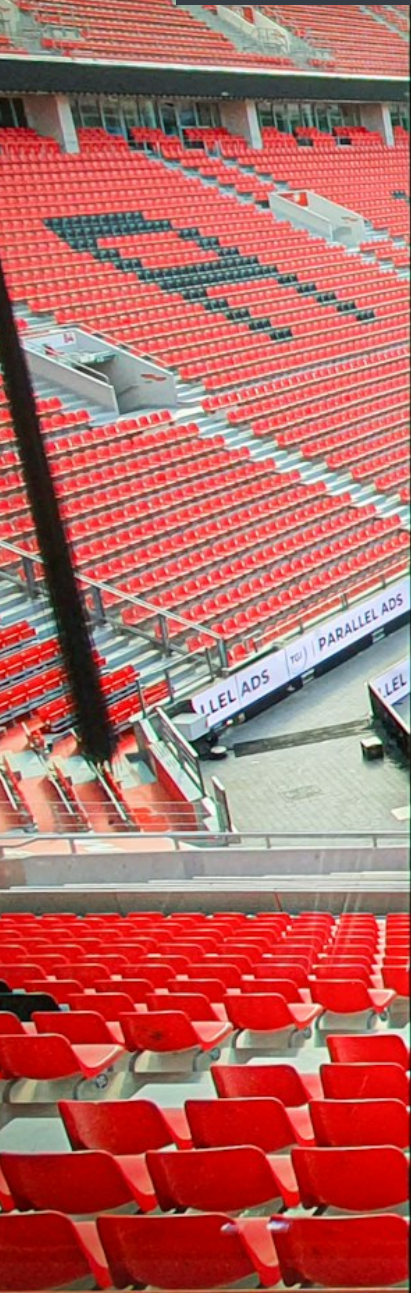
- Overall number of participants at events **was restricted**
- Certain **spacing between participants** had to be ensured
- Larger events required **vaccination certificates** for all

Question: What are the main issues for the organizers?

Main Difficulties

- Organization of larger events **is costly**
- Even without a pandemic a **financial risk**
- **Administrative Burden** for vaccination certificates
- Reduced capacity is a **loss of revenue**
- Implementing and enforcing **distancing rules**
- Managing **different priorities** of groups

Idea: Optimizing Seating Plans



Background

- **Applications:** sport arenas, concert halls, movie theaters, lecture halls, etc.
- People from the same group are **seated together**
- Venues have **specific seating**, season tickets, VIPs, etc.

! Optimizing seating plans can help to **maximize revenue** while ensuring distancing rules and other constraints are met.

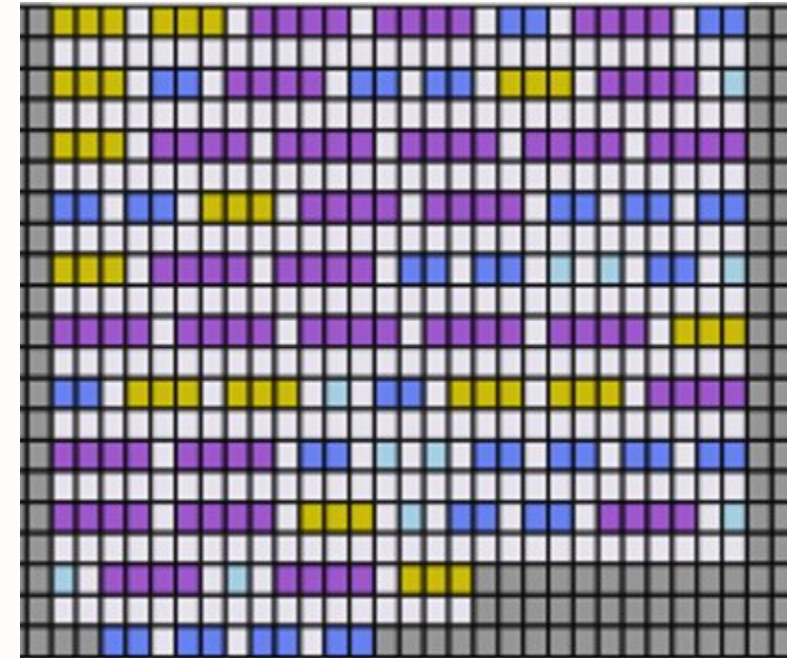
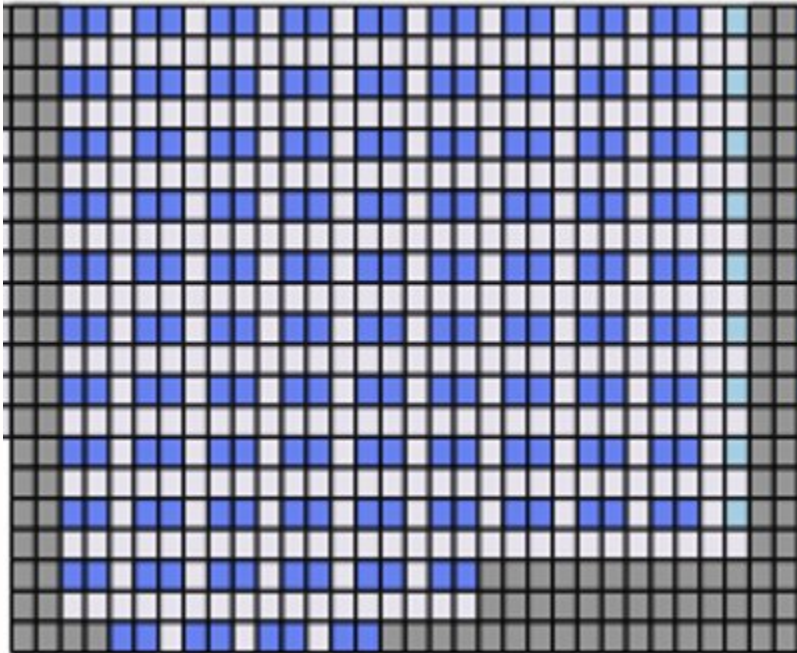
Learning Goals

After this lecture, you will be able to:

- **Formulate** the arena seating problem as a binary program
- **Model** group placements with one binary variable per starting seat
- **Enforce** distancing rules with rectangle covering constraints
- **Distinguish** operational, tactical, and strategic seating decisions

Problem Structure

Example: Two different plans



Different Approaches Possible

1. Operational
2. Tactical
3. Strategic

Question: What are these approaches in general and how do they relate to arena seating?

Operational

- **Short-term**, day-to-day decisions
- Focused on **immediate execution**

Question: What is an example of this approach?

- Given **tomorrow's demand** of differently sized groups
- Score groups (importance, sponsors, VIP, season ticket,...)
- Assigning specific seats for **tomorrow's event**

Tactical

- **Medium-term** planning (weeks to months)
- Bridges operational and strategic levels

Question: What is an example of this approach?

- Given distribution of **expected demand** for groups
- Score groups (importance, sponsors, VIP, season ticket,...)
- Plan seating arrangements for an **upcoming season**

Strategic

- Long-term planning (months to years)
- Focus on overall goals and policies

Question: What is an example of this approach?

- Design one **fixed layout** that works well across different demand scenarios
- Maximize the overall **space utilization**
- Sell the resulting layout as a **standard product** on the market

Main Question

Task: Fill the seating area given distancing regulations and venue-specific constraints.

Question: Any ideas on how to approach this?



Knapsack

Knapsack Problem

Knapsack Problem in 2D

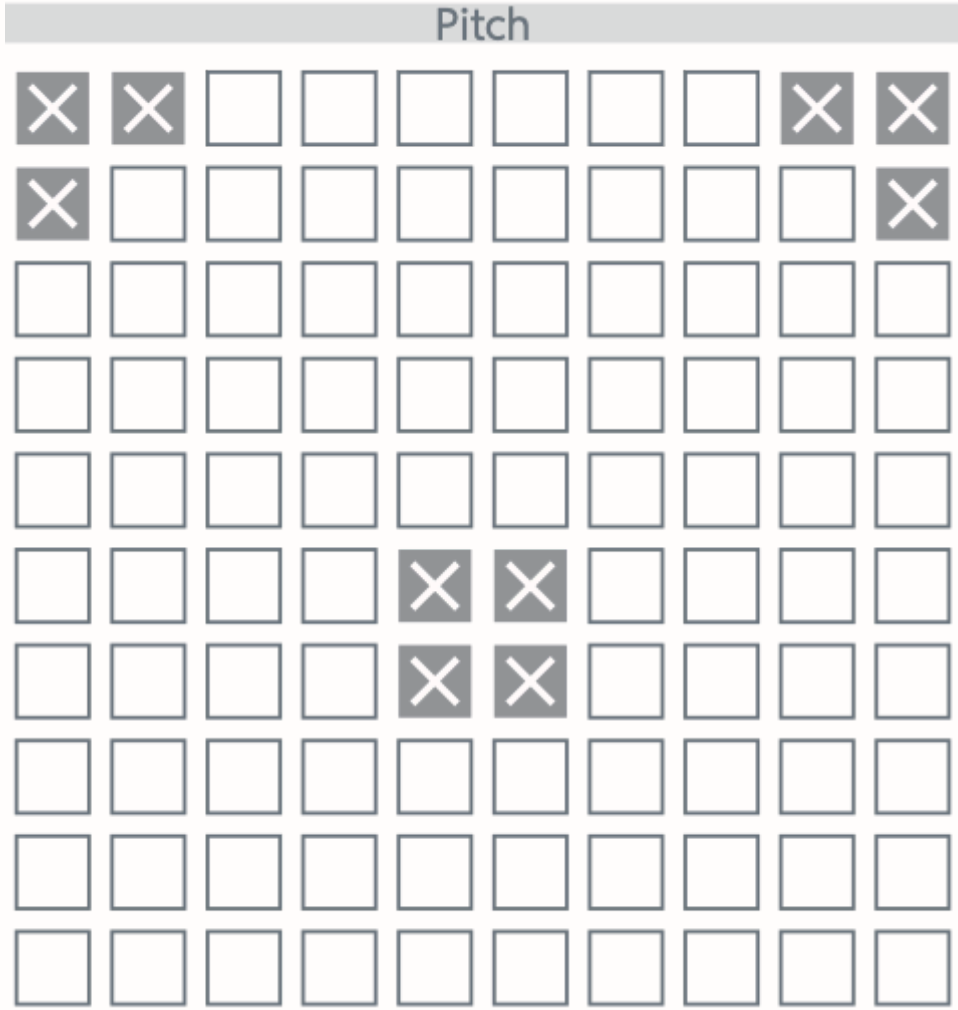
Adaptation to Seating

- Horizontal dimension to place groups of participants
- Vertical dimension to ensure enough spacing between rows
- Maximize the “value” of the allocated groups
- Value can be the number of seats or a score

 Idea behind the model was developed by [Dr. Matthes Koch](#).

Your Turn

Seat groups for as many points as possible, with an empty seat beside, in front of and diagonally to every group and at most 2 groups per row. Drag a group onto the seats, its left end where you let go; drag it off the stand to take it out.



a	1	x3
b	2	x2
c	4	x3
d	4	x5
e	5	x2
f	6	x1
g	12	x1

Drag a group onto a seat, or tap a group, then a seat.



You 0 points

Ready: press
Optimize to compare
with the best plan.

Optimize

Reset

New

Which Groups Did You Pick First?

Question: How valuable is each group per seat?

- Groups c and g yield **2 points per seat**
- Group e yields 1.25 points, all other groups exactly 1
- Grabbing the “densest” groups first is a **greedy strategy**

 Placing all groups would require 53 seats and yield 67 points — but the blocked seats and the distancing rules make this impossible.

How Good Is Your Plan?

Question: What is the best possible score?

- The optimal seating plan reaches **52 points**
- Manual plans usually fall a few points short
- Greedy choices are a good start, but **combining** them optimally is hard



In the tutorial, we will model exactly this seating area and let the solver find the optimal plan **within seconds**.

Model Formulation

Sets?

Question: What could be the sets?

- $\mathcal{G}$ - Set of groups, indexed by g
- $\mathcal{R}$ - Set of rows, indexed by r
- $\mathcal{C}$ - Set of columns, indexed by c
- $\mathcal{C}_{g,r}$ - Available seats of row r for group g , indexed by c

i $\mathcal{C}_{g,r}$ contains only **feasible starting seats**: a seat c belongs to it only if the entire group g fits into row r from seat c onward without crossing a blocked seat or the end of the row. This is why the set depends on both g and r .

Parameters?

Question: What could be possible parameters?

- p_r - Maximal number of groups allowed in one row r
- d_g - Required seats of a group g in a row
- h - Safety distance between groups sitting next to each other
- b - Vertical safety distance between groups
- v_g - Value of an allocation of the group g

Variables & Objective

Decision Variable?

! Our goal is to:

Maximize the group values by filling the seating area given distancing regulations between groups and venue-specific constraints.



Each group is represented by **one binary variable**. We don't need to block each seat explicitly with a binary variable! The sets $\mathcal{C}_{g,r}$ make this possible, as variables only exist where the **whole group** actually fits.

Decision Variable

i We need the following sets:

- All the groups, $g \in \mathcal{G}$
- All the rows, $r \in \mathcal{R}$
- All the columns, $c \in \mathcal{C}$

Question: What could be our decision variable?

- $X_{g,r,c}$ - 1, if **first left seat** of g is assigned to r in c , else 0

Objective Function?

! Our main objective is to:

Maximize the group values by filling the seating area given distancing regulations between groups and venue-specific constraints.

Question: How again are groups allocated?

- By the allocation of the **first left seat** of a group to a row and column in the seating area

Objective Function

i We need the following parameters and variables:

- v_g - Value of an allocation of the group g
- $X_{g,r,c}$ - 1, if **first left seat** of g is assigned to r in c , else 0

Question: What could be our objective function?

$$\text{Maximize } \sum_{g \in \mathcal{G}} \sum_{r \in \mathcal{R}} \sum_{c \in \mathcal{C}_{g,r}} v_g \times X_{g,r,c}$$

Constraints

Necessary Constraints

Question: What constraints do we need?

- Assign each group only once
- Restrict the number of groups in each row
- Ensure the horizontal social distance
- Keep the vertical social distance

Assign Each Group Only Once?

! The goal of this constraint is to:

Ensure that each group is allocated only once in the entire seating area.

i We need the following:

- $X_{g,r,c}$ - 1, if first left seat of g is assigned to r in c , else 0
- $\mathcal{G}$ - Set of groups, indexed by g
- $\mathcal{R}$ - Set of rows, indexed by r
- $\mathcal{C}_{g,r}$ - Set of columns of row r for group g , indexed by c

Assign Each Group Only Once

Question: What could be the constraint?

$$\sum_{r \in \mathcal{R}} \sum_{c \in \mathcal{C}_{g,r}} X_{g,r,c} \leq 1 \quad \forall g \in \mathcal{G}$$

 This “set packing” constraint ensures that a group is only **assigned once**.

Restrict Groups Per Row?

! The goal of this constraint is to:

Ensure that the number of groups in each row does not exceed the maximum allowed number of groups.

i We need the following:

- p_r - Maximal number of groups allowed in one row r
- $X_{g,r,c}$ - 1, if first left seat of g is assigned to r in c , else 0

Restrict Groups Per Row

Question: What could be the constraint?

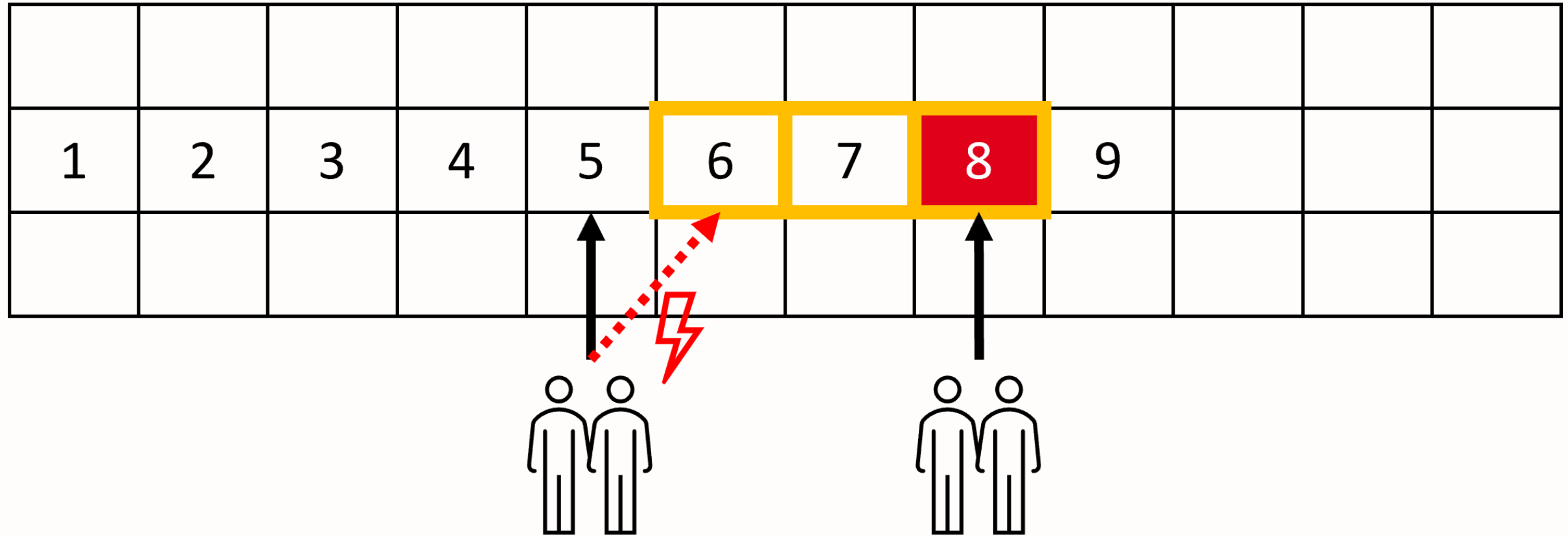
$$\sum_{g \in \mathcal{G}} \sum_{c \in \mathcal{C}_{g,r}} X_{g,r,c} \leq p_r \quad \forall r \in \mathcal{R}$$

- i* We want to place as **many highly scoring groups as possible**, but people need to move to buy drinks or use the restroom. Depending on the venue, they should **not cross other groups** in the **same row**.

The last two
constraints are
somewhat tricky!

Social Distancing

Central Idea



Assume **one seat between groups must be kept empty**. If one group takes seat 8, it uses seats 8 and 9. We thus **cannot allocate** another group of size 2 to seats 6, 7 or 8.

Horizontal Social Distance?

Question: Any ideas how to implement this?

! The goal of this constraint is to:

Ensure that the horizontal social distance is maintained between groups.

i We need the following:

- $X_{g,r,c}$ - 1, if first left seat of g is assigned to r in c , else 0
- d_g - Required seats of group g in a row
- h - Safety distance between groups sitting next to each other

Horizontal Social Distance

As the constraint is based on a rather complex set, you don't have to find it on your own.

$$\sum_{g \in \mathcal{G}} \sum_{\tilde{c} \in \tilde{\mathcal{C}}_{g,c}} X_{g,r,\tilde{c}} \leq 1 \quad \forall r \in \mathcal{R}, c \in \mathcal{C}$$

i At first glance, this constraint **looks rather easy**, but it is not - it is based on the set $\tilde{\mathcal{C}}_{g,c}$, a set not yet defined in the lecture.

The Social Distancing Set

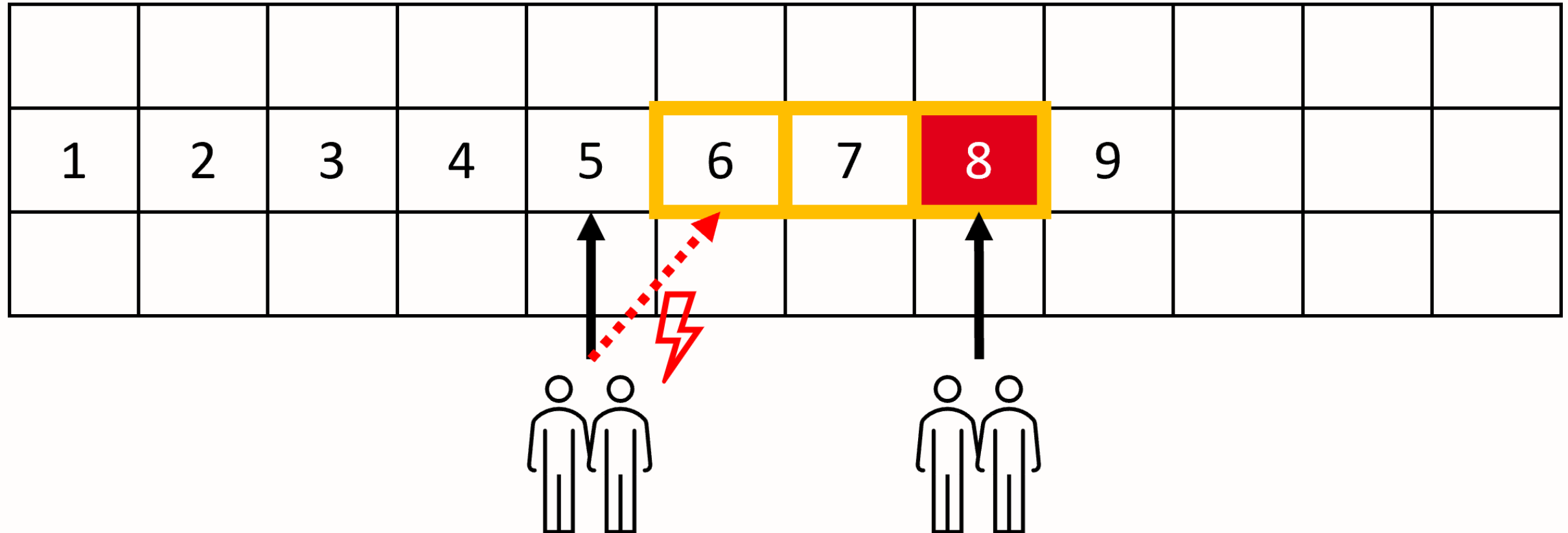
$$\tilde{\mathcal{C}}_{g,c} = \{\tilde{c} \in \mathcal{C}_{g,r} \mid c - d_g + 1 - h \leq \tilde{c} \leq c\}$$

Remember:

- d_g - Required seats of group g in a row
- h - Safety distance between groups sitting next to each other
- $\mathcal{C}_{g,r}$ - Feasible starting seats, as variables only exist for these seats (the row r follows from the constraint)

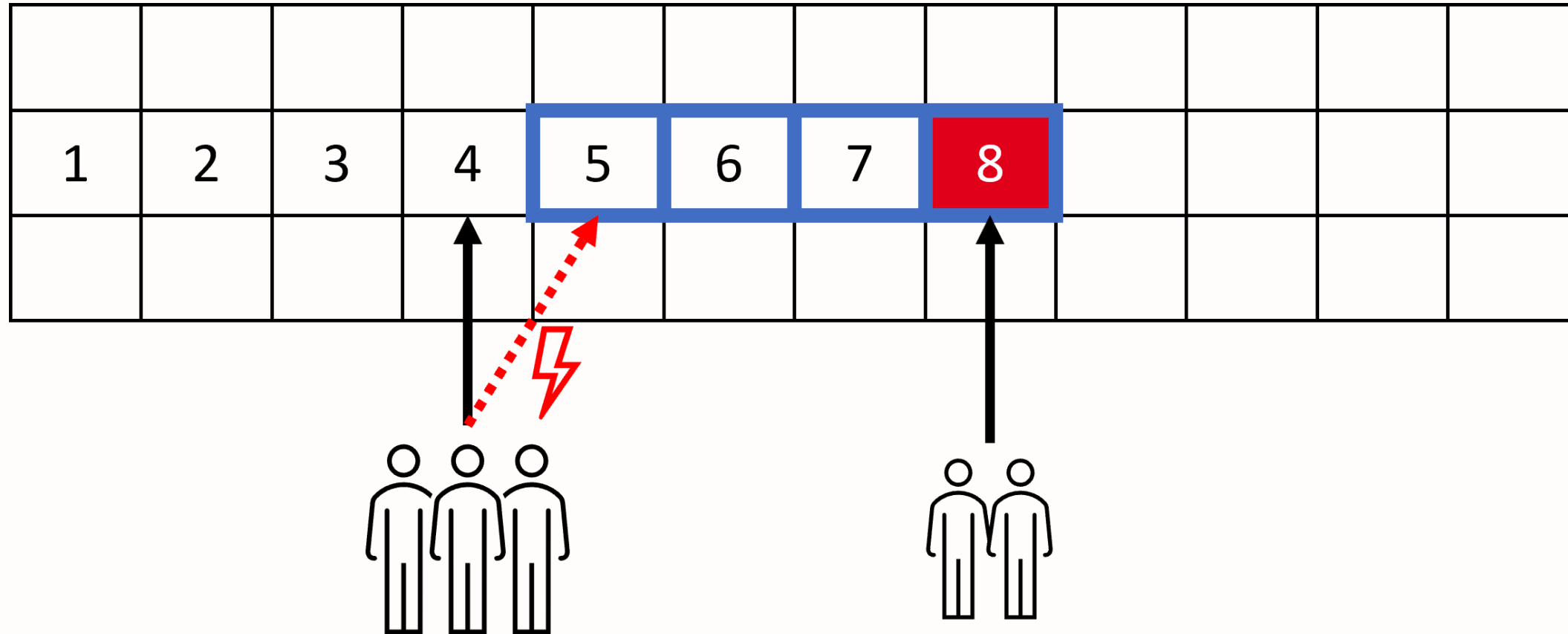
Question: Can anybody explain the set?

Example: Two Groups



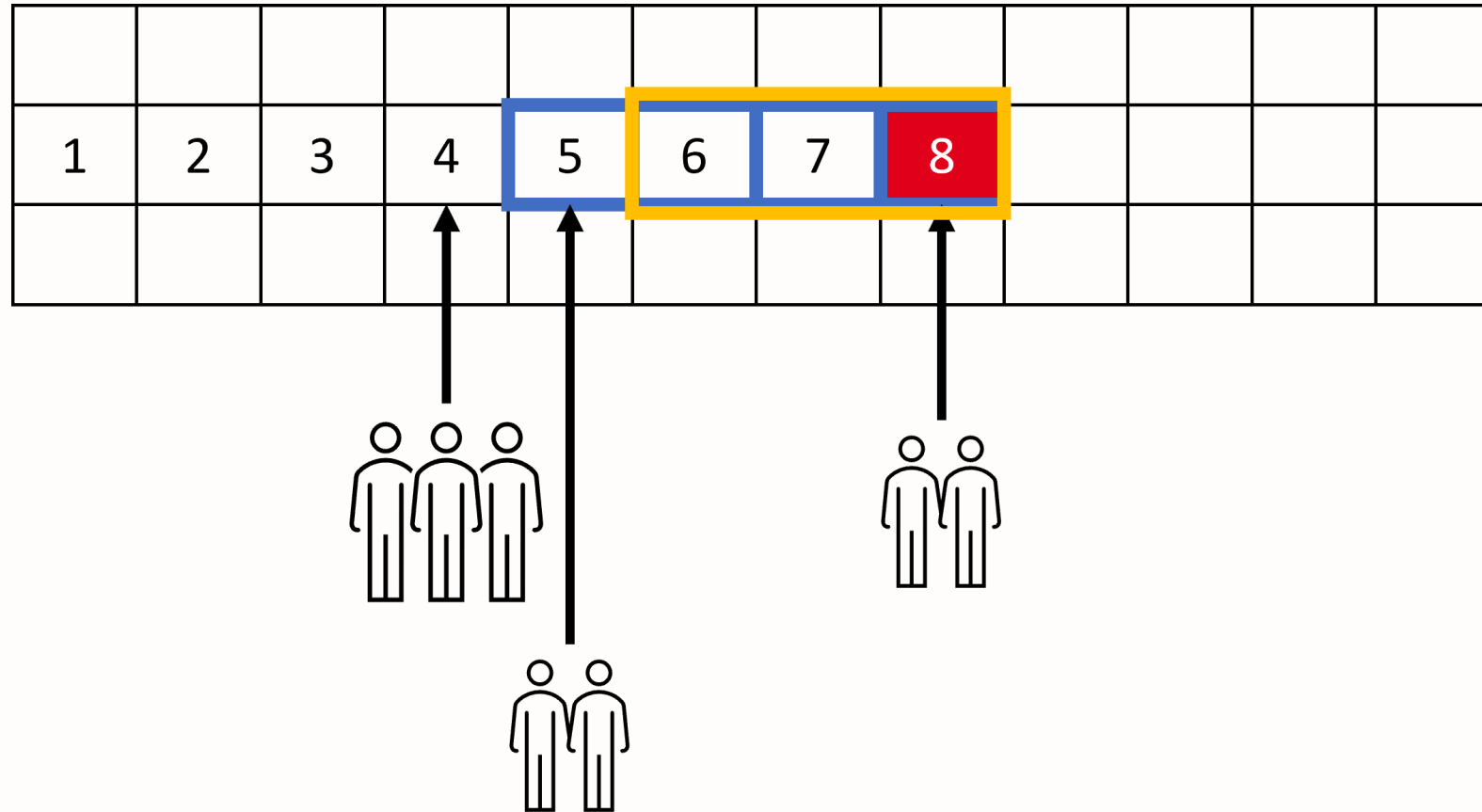
$$\underbrace{X_{1,2,6} + X_{1,2,7} + X_{1,2,8}}_{g=1} + \underbrace{X_{2,2,6} + X_{2,2,7} + X_{2,2,8}}_{g=2} \leq 1 \quad (r = 2, c = 8)$$

Example: Different Group Sizes



$$\underbrace{X_{1,2,6} + X_{1,2,7} + X_{1,2,8}}_{g=1} + \underbrace{X_{2,2,5} + X_{2,2,6} + X_{2,2,7} + X_{2,2,8}}_{g=2} \leq 1 \quad (r = 2, c = 8)$$

Example: Three Groups



$$\underbrace{X_{1,2,6} + X_{1,2,7} + X_{1,2,8}}_{g=1} + \underbrace{X_{2,2,6} + X_{2,2,7} + X_{2,2,8}}_{g=2} + \underbrace{X_{3,2,5} + X_{3,2,6} + X_{3,2,7} + X_{3,2,8}}_{g=3} \leq 1 \quad (r = 2, c = 8)$$

Do you see
the pattern?

Vertical Social Distance?

! The goal of this constraint is to:

Ensure that the vertical social distance is maintained between groups.

i We need the following:

- b - Vertical safety distance between groups
- $X_{g,r,c}$ - 1, if first left seat of g is assigned to r in c , else 0

Vertical Social Distance

Question: What could be the constraint?



It is an extension of the horizontal social distance constraint we used before, but now we **block a rectangular area** instead of a single row.

$$\sum_{g \in \mathcal{G}} \sum_{\tilde{r} \in \tilde{\mathcal{R}}_r} \sum_{\tilde{c} \in \tilde{\mathcal{C}}_{g,c}} X_{g,\tilde{r},\tilde{c}} \leq 1 \quad \forall r \in \mathcal{R}, c \in \mathcal{C}$$

Vertical Distance Set

Question: What could be the set?

$$\tilde{\mathcal{R}}_r = \{\tilde{r} \in \mathcal{R} \mid r - b \leq \tilde{r} \leq r\}$$

i Remember:

- b - Vertical safety distance between groups
- $X_{g,r,c}$ - 1, if first left seat of g is assigned to r in c , else 0

Let's look at an **example**.

Example: Two Groups

1	2	3	4	5	6	7	8		

Example placement



1	2	3	4	5	6	7	8		

1	2	3	4	5	6	7	8		

- Yellow seats are **blocked by the group** in row 3 and column 8
- Blue allocations are possible (if second group has **size 2**)

Arena Seating Problem

$$\text{Maximize} \quad \sum_{g \in \mathcal{G}} \sum_{r \in \mathcal{R}} \sum_{c \in \mathcal{C}_{g,r}} v_g \times X_{g,r,c}$$

subject to:

$$\sum_{r \in \mathcal{R}} \sum_{c \in \mathcal{C}_{g,r}} X_{g,r,c} \leq 1 \quad \forall g \in \mathcal{G}$$

$$\sum_{g \in \mathcal{G}} \sum_{c \in \mathcal{C}_{g,r}} X_{g,r,c} \leq p_r \quad \forall r \in \mathcal{R}$$

$$\sum_{g \in \mathcal{G}} \sum_{\tilde{r} \in \tilde{\mathcal{R}}_r} \sum_{\tilde{c} \in \tilde{\mathcal{C}}_{g,c}} X_{g,\tilde{r},\tilde{c}} \leq 1 \quad \forall r \in \mathcal{R}, c \in \mathcal{C}$$

$$X_{g,r,c} \in \{0, 1\} \quad \forall g \in \mathcal{G}, r \in \mathcal{R}, c \in \mathcal{C}_{g,r}$$

Why No Horizontal Constraint?

Question: Why does the model contain only three constraints?

- The vertical set $\tilde{\mathcal{R}}_r$ also contains $\tilde{r} = r$, the **row itself**
- For $\tilde{r} = r$, the rectangle constraint is **exactly** the horizontal one

i The rectangle constraint **subsumes** the horizontal social distance constraint. We can thus drop the horizontal constraint without losing anything.

Model Characteristics

Characteristics

Questions: On model characteristics

- Is the model formulation linear/ non-linear?
- What kind of variable domains do we have?

 The model is **linear**, as the objective and all constraints are weighted sums of the variables. All variables are **binary**, making it a pure binary program.

Model Assumptions

Questions: On model assumptions

- What assumptions have we made?
- Is our approach strategic or tactical/operational?
- Have we considered all social distancing constraints?
- What about aisle seats?
- Can you think of any other real-world constraints?

In Practice

Arena Seating Optimization



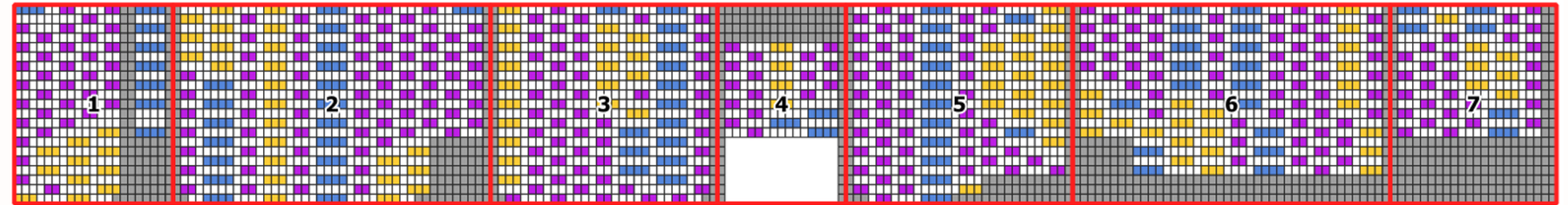
Case study VfL Osnabrück

- Relegation Return Match in 2021
- 241 additional seats allocated (+12 percent)
- Compliance with all distancing requirements
- Approval from authorities

! Estimated additional revenue of **8,435 EUR** for one match.

Seating Plan

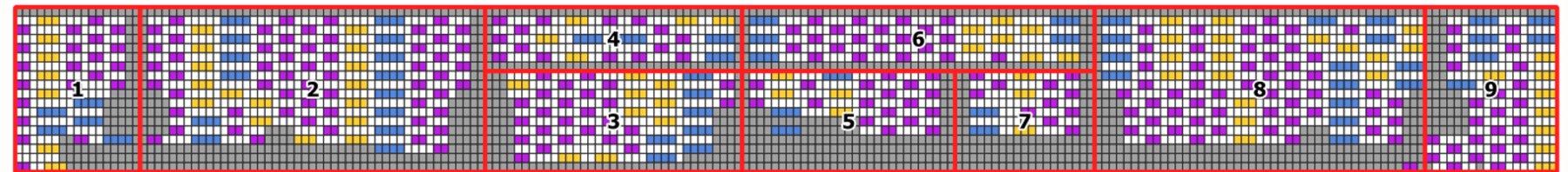
Block	Plätze		
	Verfügbar	Belegt	Belegt %
Nord 1	349	135	38.7%
Nord 2	788	290	36.8%
Nord 3	586	217	37.0%
Nord 4	150	56	37.3%
Nord 5	549	204	37.2%
Nord 6	692	254	36.7%
Nord 7	266	99	37.2%
Nord Gesamt	3380	1255	37.1%
Süd 1	214	83	38.8%
Süd 2	587	218	37.1%
Süd 3	281	105	37.4%
Süd 4	165	61	37.0%
Süd 5	153	60	39.2%
Süd 6	225	83	36.9%
Süd 7	105	39	37.1%
Süd 8	576	213	37.0%
Süd 9	251	96	38.2%
Süd Gesamt	2557	958	37.5%
Nord+Süd Gesamt	5937	2213	37.3%



Nordtribüne



Südtribüne



Modellrechnung Bremer Brücke

Related Work

- The model of this lecture is published in Dkaidik and Koch ([2025](#))

Similar studies have been conducted globally:

- Music Hall Eindhoven: Blom et al. ([2022](#))
- US College-level venues, e.g. Football, Basketball, Hockey
- Safe Seating Solutions platform
- General 2D-Knapsack applications

Conclusion

Optimization Benefits

- Optimization enables **rapid generation and evaluation**
- We can easily adapt to **various distancing requirements**:
 - Horizontal and vertical spacing between groups
 - Groups per row limits
 - Aisle seat restrictions
 - Group size constraints
 - Multi-row group allocation

Wrap Up

- **Revenue optimization** through applied optimization
- **Increased participant capacity** vs basic approaches
- Flexible adaptation to **various distancing requirements**
- Can be adapted easily to **any seating requirements**

i And that's it for today's lecture!

We now have covered the arena seating problem based on a real-world application and are ready to start solving the corresponding tasks in the upcoming tutorial.

Questions?

Literature

Literature

- Press coverage of the VfL Osnabrück case: [University of Hamburg newsroom](#) (in German)

For more interesting literature to learn more about Julia, take a look at the [literature list](#) of this course.

Blom, Danny, Rudi Pendavingh, and Frits Spieksma. 2022. “Filling a Theater During the COVID-19 Pandemic.” *INFORMS Journal on Applied Analytics* 52 (6): 473–84.

<https://doi.org/10.1287/inte.2021.1104>.

Dkaidik, Usama, and Matthes Koch. 2025. “Arena Seat Planning Under Distancing Rules.” In *Operations Research Proceedings 2024*. Springer. https://doi.org/10.1007/978-3-031-92575-7_12.