Lecture XI - Arena Seat Planning under Distancing Rules

Applied Optimization with Julia

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Introduction

Covid-19 Pandemic

Challenges for Live Events

- Overall number of participants at events was restricted
- Certain spacing between participants had to be ensured
- Larger events required vaccination certificates for all

. . .

Question: What are the main issues for the organizers?

Main Difficulties

- Organization of larger events is costly
- Even without a pandemic a financial risk
- Administrative Burden for vaccination certificates
- Reduced capacity is a loss of revenue
- Implementing and enforcing distancing rules
- Managing different priorities of groups

Idea: Optimizing Seating Plans

Background

- Applications: sport arenas, concert halls, movie theaters, lecture halls, etc.
- People from the same group are seated together
- Venues have specific seating, season tickets, VIPs, etc.

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> ! Important
>
> Optimizing seating plans can help to maximize revenue while ensuring distancing rules and other constraints are met.

Problem Structure

Example: Two different plans

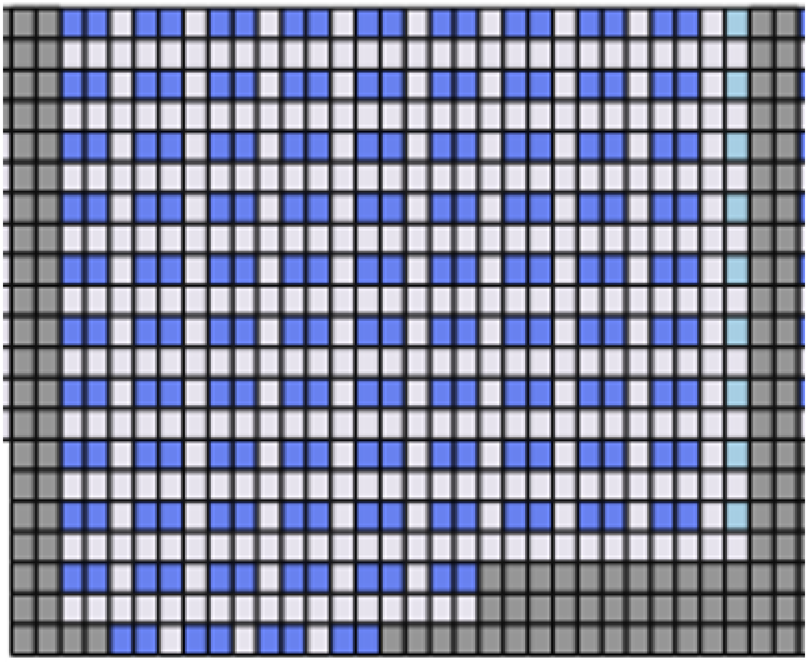


Figure 1: Fixed double-seat layout

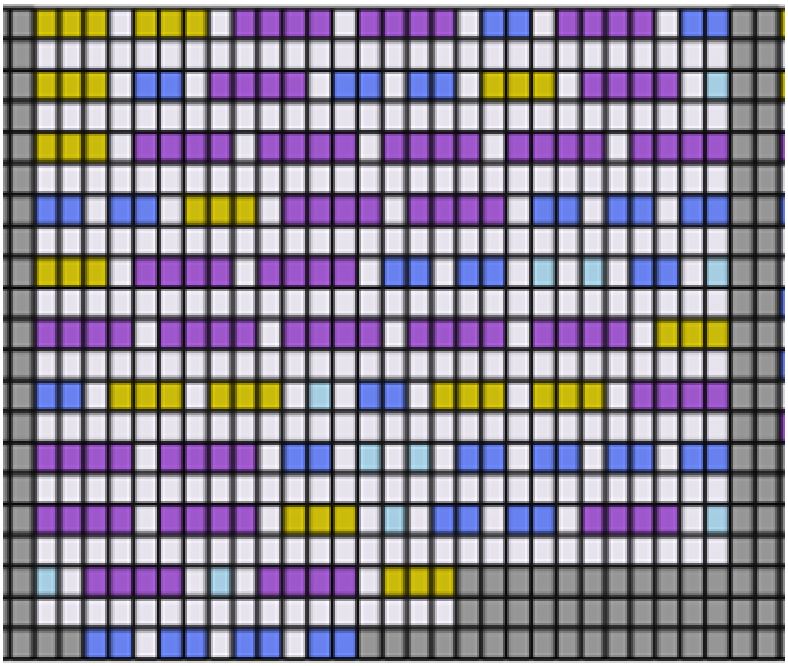


Figure 2: Flexible group-value layout

Different Approaches Possible

1. Operational

2. Tactical
3. Strategic

. . .

Question: What are these approaches in general and how do they relate to arena seating?

Operational

- Short-term, day-to-day decisions
- Focused on immediate execution

. . .

Question: What is an example for this approach?

. . .

- Given tomorrow's demand of differently sized groups
- Score groups (importance, sponsors, VIP, season ticket,…)
- Assigning specific seats for tomorrow's event

Tactical

- Medium-term planning (weeks to months)
- Bridges operational and strategic levels

. . .

Question: What is an example for this approach?

. . .

- Given distribution of expected demand for groups
- Score groups (importance, sponsors, VIP, season ticket,…)
- Plan seating arrangements for an upcoming season

Strategic

- Long-term planning (months to years)
- Focus on overall goals and policies

. . .

Question: What is an example for this approach?

. . .

- Designing flexible seating layouts that work for scenarios
- Maximize the overall space utilization
- Sell the resulting maximized seating pattern on market

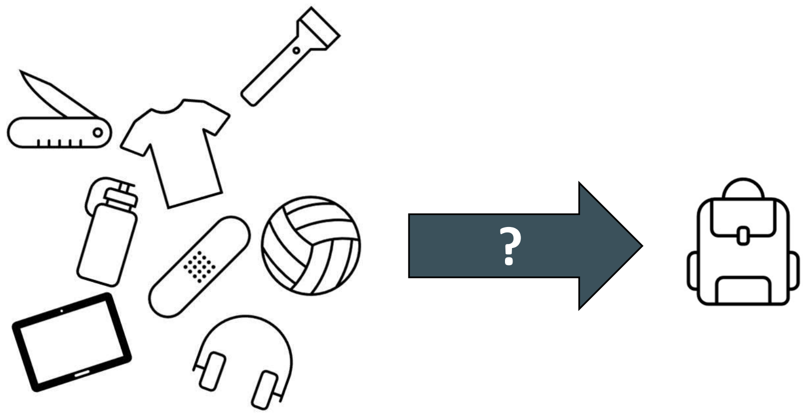
Main Question

Task: Fill the seating area given distancing regulations and venue-specific constraints.

Question: Any ideas on how to approach this?

Knapsack

Knapsack Problem



- Standard model in Operations Research
- Select items from a pool under capacity constraints

Knapsack Problem in 2D

	c1	c2	c3	c4	c5	c6	c7	c8	c9	c10	c11	c12	c13	c14	c15	c16	c17	c18	c19	c20	c21	c22	c23	c24	c25	c26	c27	c28	c29	c30
r1	7	7	7	7	7	7	7	7	7	7	7	7	7	7	8	8	8	8	8	8	8	8	8	8	8	8	8	8	12	12
r2	7	7	7	7	7	7	7	7	7	7	7	7	7	7	8	8	8	8	8	8	8	8	8	8	8	8	8	8	12	12
r3	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	3	3	3	3	3	4	4	4	4	4	12	12
r4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	3	3	3	3	3	4	4	4	4	4	12	12
r5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	3	3	3	3	3	4	4	4	4	4	12	12
r6	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	3	3	3	3	3	4	4	4	4	4	12	12
r7	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	3	3	3	3	3	4	4	4	4	4	12	12
r8	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	3	3	3	3	3	4	4	4	4	4	12	12
r9	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	3	3	3	3	3	4	4	4	4	4	12	12
r10	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	3	3	3	3	3	4	4	4	4	4	12	12
r11	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	3	3	3	3	3	4	4	4	4	4	12	12
r12	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	3	3	3	3	3	4	4	4	4	4	12	12
r13	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	3	3	3	3	3	4	4	4	4	4	12	12
r14	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	3	3	3	3	3	4	4	4	4	4	12	12
r15	1	1	1	9	9	9	9	9	9	9	9	9	9	9	9	9	9		3	3	3	3	3	4	4	4	4	4		
r16	1	1	1	9	9	9	9	9	9	9	9	9	9	9	9	9	9	2	2	2	2	2	2	2	2	2	2	2	2	2
r17	1	1	1	10	10	10	10	10	10	10	10	10	10	10	10	10	10	2	2	2	2	2	2	2	2	2	2	2	2	2
r18	1	1	1	10	10	10	10	10	10	10	10	10	10	10	10	10	10	2	2	2	2	2	2	2	2	2	2	2	2	2
r19	1	1	1	11	11	11	11	11	11	11	11	11	11	11	11	11	11	2	2	2	2	2	2	2	2	2	2	2	2	2
r20	1	1	1	11	11	11	11	11	11	11	11	11	11	11	11	11	11	2	2	2	2	2	2	2	2	2	2	2	2	2

- Now, Items block space in 2D, as illustrated here

Adaption to Seating

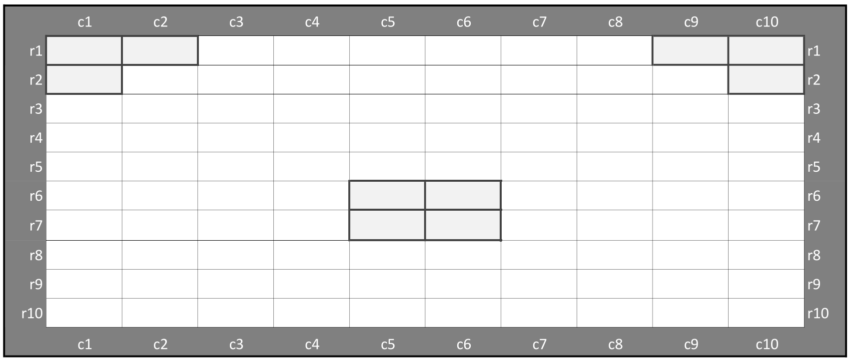
- Horizontal dimension to place groups of participants
- Vertical dimension to ensure enough spacing between rows
- Maximize the “value” of the allocated groups
- Value can be the number of seats or a score

. . .

> **ℹ Note**
>
> Idea behind the model was developed by Dr. Matthes Koch.

Hands-on Exercise

Task: Allocate as many high-value groups as possible.



Available Groups

Grouptype	Req. Seats	Score	Available	Allocated	Value
a	1	1	3		
b	2	2	2		
c	2	4	3		
d	4	4	5		
e	4	5	2		
f	6	6	1		
g	6	12	1		
Total					

Seating Constraints

- 1 empty seat between groups
- 1 empty seat front-to-back
- 1 empty seat diagonally
- Only 2 groups per row are allowed
- Grey seats represent obstacles

. . .

You have 5 minutes to find a solution.

Question: What is your total score?

Model Formulation

Sets?

Question: What could be the sets?

...

- $\mathcal{G}$ - Set of groups, indexed by g
- $\mathcal{R}$ - Set of rows, indexed by r
- $\mathcal{C}$ - Set of columns, indexed by c
- $\mathcal{C}_{g,r}$ - Available seats of row r for group g, indexed by c

...

> **Note**
>
> $\mathcal{C}_r$ ensures that we only consider unblocked seats in each row.

Parameters?

Question: What could be possible parameters?

...

- p_r - Maximal number of groups allowed in one row r
- d_g - Required seats of a group g in a row
- h - Safety distance between groups sitting next to each other
- b - Vertical safety distance between groups
- v_g - Value of an allocation of the group g

Variables and Objective

Decision Variable?

> **! Our goal is to:**
>
> Maximize the group values by filling the seating area given distancing regulations between groups and venue-specific constraints.

> **Tip**
>
> Each group is represented by one binary variable. We don't need to block each seat explicitly with a binary variable!

Decision Variable

i We need the following sets:

- All the groups, $g \in \mathcal{G}$
- All the rows, $r \in \mathcal{R}$
- All the columns, $c \in \mathcal{C}$

Question: What could be our decision variable?

...

- $X_{g,r,c}$ - 1, if first left seat of g is assigned to r in c, else 0

Objective Function?

! Our main objective is to:

Maximize the group values by filling the seating area given distancing regulations between groups and venue-specific constraints.

...

Question: How again are groups allocated?

...

- By the allocation of the first left seat of a group to a row and column in the seating area

Objective Function

i We need the following parameters and variables:

- v_g - Value of an allocation of the group g
- $X_{g,r,c}$ - 1, if first left seat of g is assigned to r in c, else 0

...

Question: What could be our objective function?

...

$$\text{maximize} \quad \sum_{g \in \mathcal{G}} \sum_{r \in \mathcal{R}} \sum_{c \in \mathcal{C}_{g,r}} v_g \times X_{g,r,c}$$

Constraints

Necessary Constraints

Question: What constraints do we need?

. . .

- Assign each group only once
- Restrict the number of groups in each row
- Ensure the horizontal social distance
- Keep the vertical social distance

Assign Each Group Only Once?

! The goal of this constraint is to:

Ensure that each group is allocated only once in the entire seating area.

. . .

i We need the following:

- $X_{g,r,c}$ - 1, if first left seat of g is assigned to r in c, else 0
- $\mathcal{G}$ - Set of groups, indexed by g
- $\mathcal{R}$ - Set of rows, indexed by r
- $\mathcal{C}_{g,r}$ - Set of columns of row r for group g, indexed by c

Assign Each Group Only Once

Question: What could be the constraint?

. . .

$$\sum_{r \in \mathcal{R}} \sum_{c \in \mathcal{C}_{g,r}} X_{g,r,c} \leq 1 \quad \forall g \in \mathcal{G}$$

. . .

i Note

This "set packing" constraint ensures that a group is only assigned once.

Restrict Groups Per Row?

! The goal of this constraint is to:

Ensure that the number of groups in each row does not exceed the maximum allowed number of groups.

. . .

i We need the following:

- p_r - Maximal number of groups allowed in one row r
- $X_{g,r,c}$ - 1, if first left seat of g is assigned to r in c, else 0

Restrict Groups Per Row

Question: What could be the constraint?

. . .

$$\sum_{g \in \mathcal{G}} \sum_{c \in \mathcal{C}_{g,r}} X_{g,r,c} \leq p_r \quad r \in \mathcal{R}$$

. . .

i Note

We want to place as many highly scoring groups as possible, but people need to move to buy drinks or use restroom. Depending on the venue, they should not cross other groups in the same row.

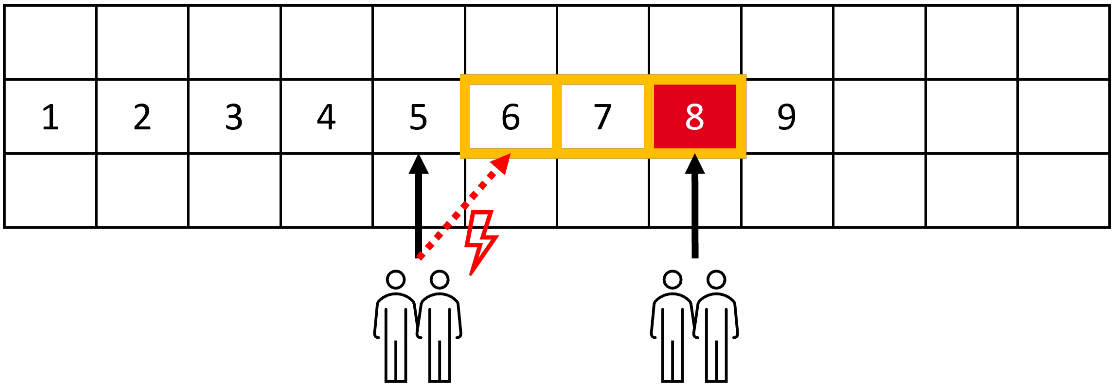
The last two

constraints are

somewhat tricky!

Social Distance Implementation

Central Idea



. . .

Tip

Assume one seat between groups must be kept empty. If one group takes seat 8, it uses seats 8 and 9. We thus cannot allocate another group of size 2 to seats 6, 7 or 8.

Horizontal Social Distance?

Question: Any ideas how to implement this?

. . .

! The goal of this constraint is to:

Ensure that the horizontal social distance is maintained between groups.

. . .

i We need the following:

- $X_{g,r,c}$ - 1, if first left seat of g is assigned to r in c, else 0
- d_g - Required seats of group g in a row
- h - Safety distance between groups sitting next to each other

---

Horizontal Social Distance

As the constraint is based on a rather complex set, you don't have to find it by yourself.

. . .

$$\sum_{g \in \mathcal{G}} \sum_{\tilde{c} \in \tilde{\mathcal{C}}_{c,g}} X_{g,r,\tilde{c}} \leq 1 \quad \forall r \in \mathcal{R}, c \in \mathcal{C}$$

...

Note

At first glance, this constraint looks rather easy, but it is not - it is based on the set $\mathcal{C}_{c,g}$ not defined yet in the lecture.

The Social Distancing Set

$$\tilde{\mathcal{C}}_{c,g} = \{\tilde{c} \in \mathcal{C} \mid c - d_g + 1 - h \leq \tilde{c} \leq c\}$$

...

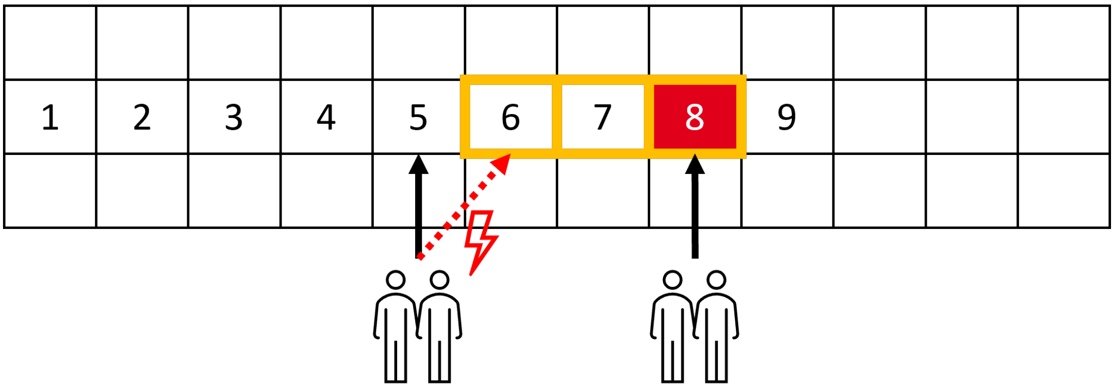
Remember:

- d_g - Required seats of group g in a row
- h - Safety distance between groups sitting next to each other

...

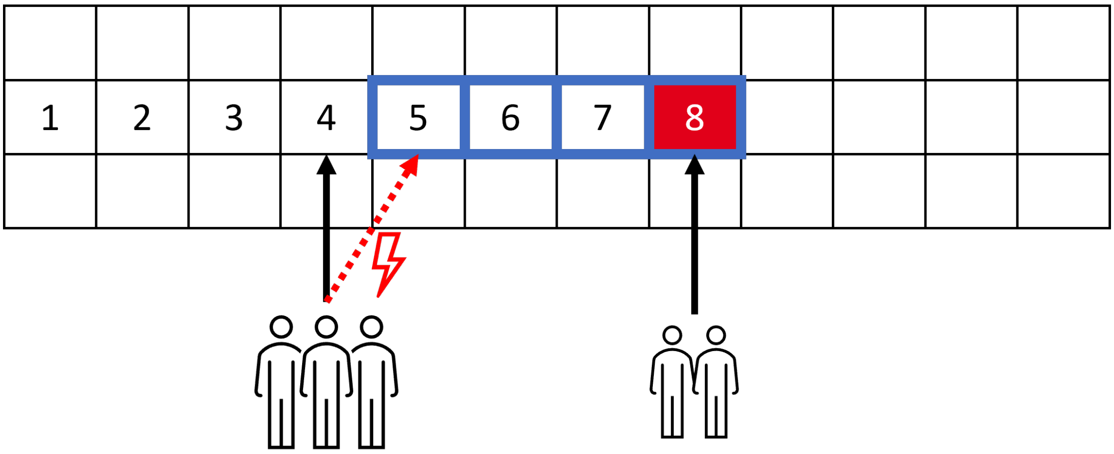
Question: Can anybody explain the set?

Example: Two Groups



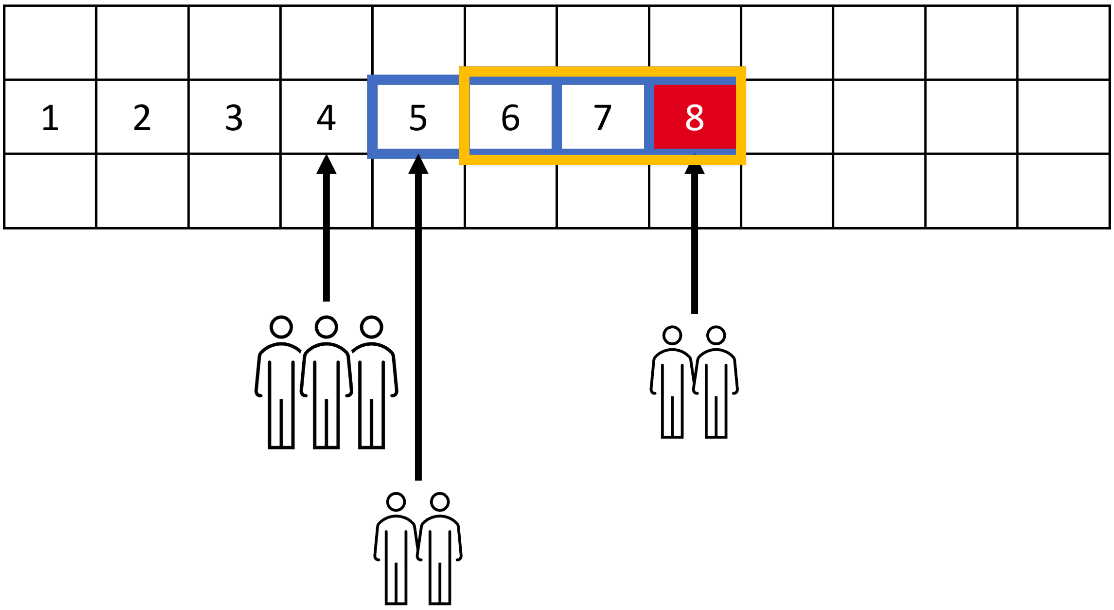
$$\underbrace{X_{1,2,\mathbf{6}} + X_{1,2,\mathbf{7}} + X_{1,2,\mathbf{8}}}_{g=1} + \underbrace{X_{2,2,\mathbf{6}} + X_{2,2,\mathbf{7}} + X_{2,2,\mathbf{8}}}_{g=2} \leq 1 \quad (r = 2, c = 8)$$

Example: Different Group Sizes



$$\underbrace{X_{1,2,\mathbf{6}} + X_{1,2,\mathbf{7}} + X_{1,2,\mathbf{8}}}_{g=1} + \underbrace{X_{2,2,\mathbf{5}} + X_{2,2,\mathbf{6}} + X_{2,2,\mathbf{7}} + X_{2,2,\mathbf{8}}}_{g=2} \leq 1 \quad (r = 2, c = 8)$$

Example: Three Groups



$$\underbrace{X_{1,2,\mathbf{6}} + X_{1,2,\mathbf{7}} + X_{1,2,\mathbf{8}}}_{g=1} + \underbrace{X_{2,2,\mathbf{6}} + X_{2,2,\mathbf{7}} + X_{2,2,\mathbf{8}}}_{g=2} + \underbrace{X_{3,2,\mathbf{5}} + X_{3,2,\mathbf{6}} + X_{3,2,\mathbf{7}} + X_{3,2,\mathbf{8}}}_{g=3} \leq 1 \quad (r = 2, c = 8)$$

Do you see
the pattern?

Vertical Social Distance?

! The goal of this constraint is to:

Ensure that the vertical social distance is maintained between groups.

. . .

i We need the following:

- b - Vertical safety distance between groups
- $X_{g,r,c}$ - 1, if first left seat of g is assigned to r in c, else 0

Vertical Social Distance

Question: What could be the constraint?

. . .

Tip

It is an extension of the horizontal social distance constraint we used before, but now we block a rectangular area instead of a single row.

. . .

$$\sum_{g \in \mathcal{G}} \sum_{\tilde{r} \in \mathcal{R}_r} \sum_{\tilde{c} \in \tilde{\mathcal{C}}_{cg}} X_{g\tilde{r}\tilde{c}} \leq 1 \quad \forall r \in \mathcal{R}, c \in \mathcal{C}$$

Vertical Distance Set

Question: What could be the set?

. . .

$$\tilde{\mathcal{R}}_r = \{\tilde{r} \in \mathcal{R} \mid r - b \leq \tilde{r} \leq r\}$$

. . .

i Note

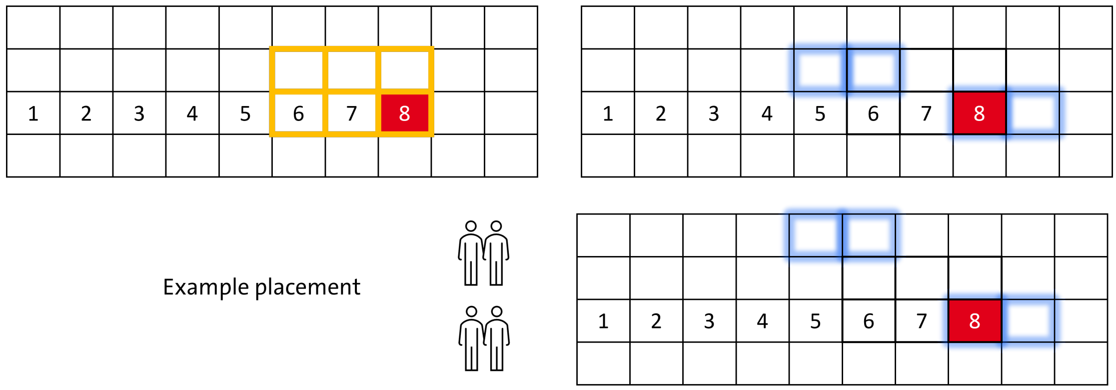
Remember:

- b - Vertical safety distance between groups
- $X_{g,r,c}$ - 1, if first left seat of g is assigned to r in c, else 0

. . .

Let's look at an example.

Example: Two Groups



- Yellow seats are blocked by the group in row 3 and column 8
- Blue allocations are possible (if second group has size 2)

Arena Seating Problem

$$\text{maximize} \quad \sum_{g \in \mathcal{G}} \sum_{r \in \mathcal{R}} \sum_{c \in \mathcal{C}_r} v_g \times X_{g,r,c}$$

subject to:

$$\sum_{r \in \mathcal{R}} \sum_{c \in \mathcal{C}_r} X_{g,r,c} \leq 1 \qquad \forall g \in \mathcal{G}$$

$$\sum_{g \in \mathcal{G}} \sum_{c \in \mathcal{C}_r} X_{g,r,c} \leq p_r \qquad \forall r \in \mathcal{R}$$

$$\sum_{g \in \mathcal{G}} \sum_{\tilde{r} \in \tilde{\mathcal{R}}_r} \sum_{\tilde{c} \in \tilde{\mathcal{C}}_{c,g}} X_{g,\tilde{r},\tilde{c}} \leq 1 \forall r \in \mathcal{R}, c \in \mathcal{C}$$

$$X_{g,r,c} \in \{0,1\} \qquad \forall g \in \mathcal{G}, \forall r \in \mathcal{R}, c \in \mathcal{C}_r$$

Model Characteristics

Characteristics

Questions: On model characteristics

- Is the model formulation linear/ non-linear?
- What kind of variable domains do we have?

Model Assumptions

Questions: On model assumptions

- What assumptions have we made?
- Is our approach strategic or tactical/operational?
- Have we considered all social distancing constraints?
- What about aisle seats?
- Can you think of any other real-world constraints?

Implementation and Impact

Arena Seating Optimization

Case study VfL Osnabrück

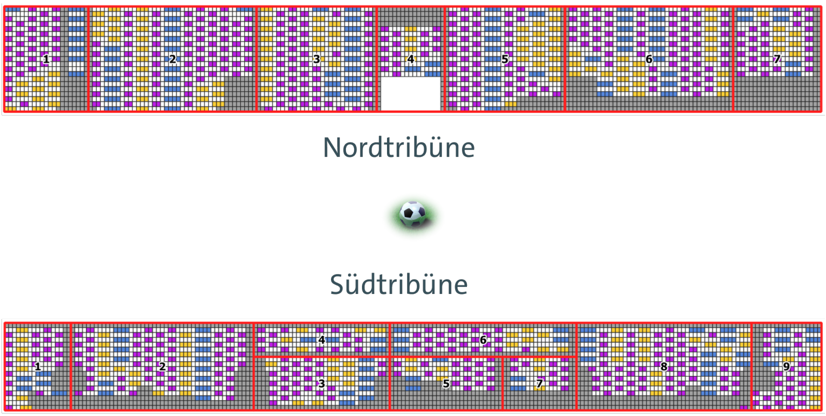
- Relegation Return Match in 2021
- 241 additional seats allocated (+12 percent)
- Compliance with all distancing requirements
- Approval from authorities

. . .

> ! Important
>
> Estimated additional revenue of 8,435 EUR for one match.

Seating Plan

	Plätze		
Block	**Verfügbar**	**Belegt**	**Belegt %**
Nord 1	349	135	38.7%
Nord 2	788	290	36.8%
Nord 3	586	217	37.0%
Nord 4	150	56	37.3%
Nord 5	549	204	37.2%
Nord 6	692	254	36.7%
Nord 7	266	99	37.2%
Nord Gesamt	**3380**	**1255**	**37.1%**
Süd 1	214	83	38.8%
Süd 2	587	218	37.1%
Süd 3	281	105	37.4%
Süd 4	165	61	37.0%
Süd 5	153	60	39.2%
Süd 6	225	83	36.9%
Süd 7	105	39	37.1%
Süd 8	576	213	37.0%
Süd 9	251	96	38.2%
Süd Gesamt	**2557**	**958**	**37.5%**
Nord+Süd Gesamt	**5937**	**2213**	**37.3%**



Modellrechnung Bremer Brücke

Related Work

Similar studies have been conducted globally:

- US College-level venues, e.g. Football, Basketball, Hockey
- Music Hall Eindhoven
- Safe Seating Solutions platform
- General 2D-Knapsack applications

Conclusion

Optimization Benefits

- Optimization enables rapid generation and evaluation
- We can easily adapt to various distancing requirements:

- Horizontal and vertical spacing between groups
- Groups per row limits
- Aisle seat restrictions
- Group size constraints
- Multi-row group allocation

Wrap Up

- Revenue optimization through applied optimization
- Increased participant capacity vs basic approaches
- Flexible adaptation to various distancing requirements
- Can be adapted easily to any seating requirements

. . .

> **i And that's it for todays lecture!**
>
> We now have covered the arena seating problem based on a real-world application and are ready to start solving the corresponding tasks in the upcoming tutorial.

Questions?

Literature

Literature I

For more interesting literature to learn more about Julia, take a look at the literature list of this course.

Bibliography