Lecture VIII - Police Districting

Applied Optimization with Julia

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Introduction

Police Service District Planning

T. Vlćek, K. Haase, M. Fliedner, and T. Cors [1]

Challenges

Question: What makes the work of emergency services complex?

- Dynamic urban development
- Changing population patterns
- Resource constraints
- Need for rapid response
- Multiple stakeholder interests

Emergency Services

Emergency services address the needs of three interest groups:

- Citizens
- Service personnel
- Administrators

Question: What could be the objectives of these groups?

Stakeholder Objectives

1. Citizens
   - Fast response times
   - Reliable service coverage
2. Service Personnel
   - Manageable workloads

- Safe working conditions

3. Administrators
   - Cost efficiency
   - Resource optimization

> **Note**
>
> Aligning the objectives of the three interest groups is challenging.

Emergency Service Districting

Question: Why might current district layouts be suboptimal?

- Many layouts date back several decades

- Often designed along highways and regions [2]
- Extensive data not used for data-driven improvement

How can we improve

this situation?

The Role of Data

Question: What data can help improve emergency services?

- Historical incident patterns
- Response time analysis
- Resource utilization metrics
- Population densities and traffic patterns

. . .

> **i** Note
>
> Extensive data collected, but often lack of tools or knowledge to leverage it.

Optimization

- Operations research (OR) models can help!
- Based on incident records and geographical information
- Improve the response of emergency services
- Help administrators in making strategic decisions
- Locate new departments or close departments [3]

Case Studies

Police Districting

> For an efficient and effective distribution of resources, police jurisdictions are divided into precincts or command districts with separate departments. These are further divided into patrol beats [4].

Service Priority Extremes

- High Priority
  - Life-threatening situations
  - Active crimes in progress
  - Multiple unit response needed
- Low Priority
  - Minor incidents

- Administrative tasks

Case Studies

- Different urban contexts
- Study of jurisdictions in
  - Germany: Large metropolitan area
  - Belgium: Large rural area
- Focus on response time optimization

...

> **Note**
>
> Part of the force patrols the streets, another part is stationed at the departments.

Dispatching

- Dispatchers assign all CFS to vehicles from the corresponding districts and patrol areas
- Officers are familiar with the area and are thus better prepared to respond appropriately [5]
- To cope with high demands, dispatchers can assign vehicles from nearby districts or beats

Potential Problem

Question: What could be the potential problem?

...

- This can lead to a domino effect
- Transferring vehicles from other districts or beats reduces coverage in those locations [6]
- This makes them vulnerable to missing resources when they need assistance themselves

Overloaded Systems

- This can lead to overloaded systems!
- Long dispatching delays due to staff shortages
- Preventive patrol hardly possible [7]
- Dispatchers constantly draw on patrol resources
- Reduces the response time of emergency services

...

> **Warning**
>
> This is a common problem in many emergency services.


Response Time
Call Length
Dispatch Time
Driving Time

- Central criterion to measure the effectiveness of emergency services is the response time
- Time between a call for aid and the arrival at the incident location
- Low response time increases the likelihood of helping and improves confidence [5]

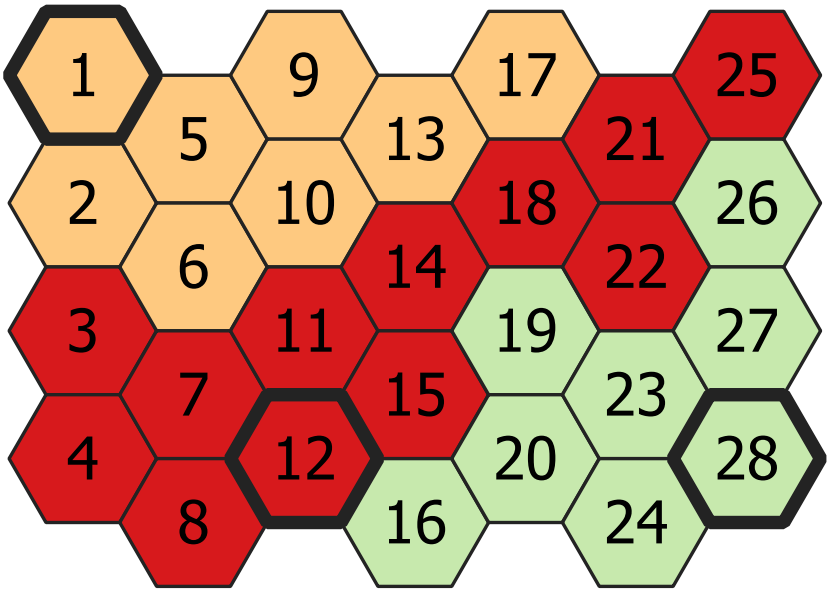
Response Time Influencers

Question: What affects response time?

- Initial contact
- Information gathering
- Unit assignment
- Resource coordination
- Route to location
- Traffic conditions

Territory Design Problem

Territory Design Problem



> Aggregation of small geographic areas, called basic areas (BAs), into geographic clusters, called districts, so that these are acceptable according to pre-defined planning criteria[1].

Objective

Question: What could be the objective?

[1] A. A. Zoltners and P. Sinha [8]

. . .

- Minimize the response time to help citizens faster while increasing the confidence in the service

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Question: What could be further objectives?

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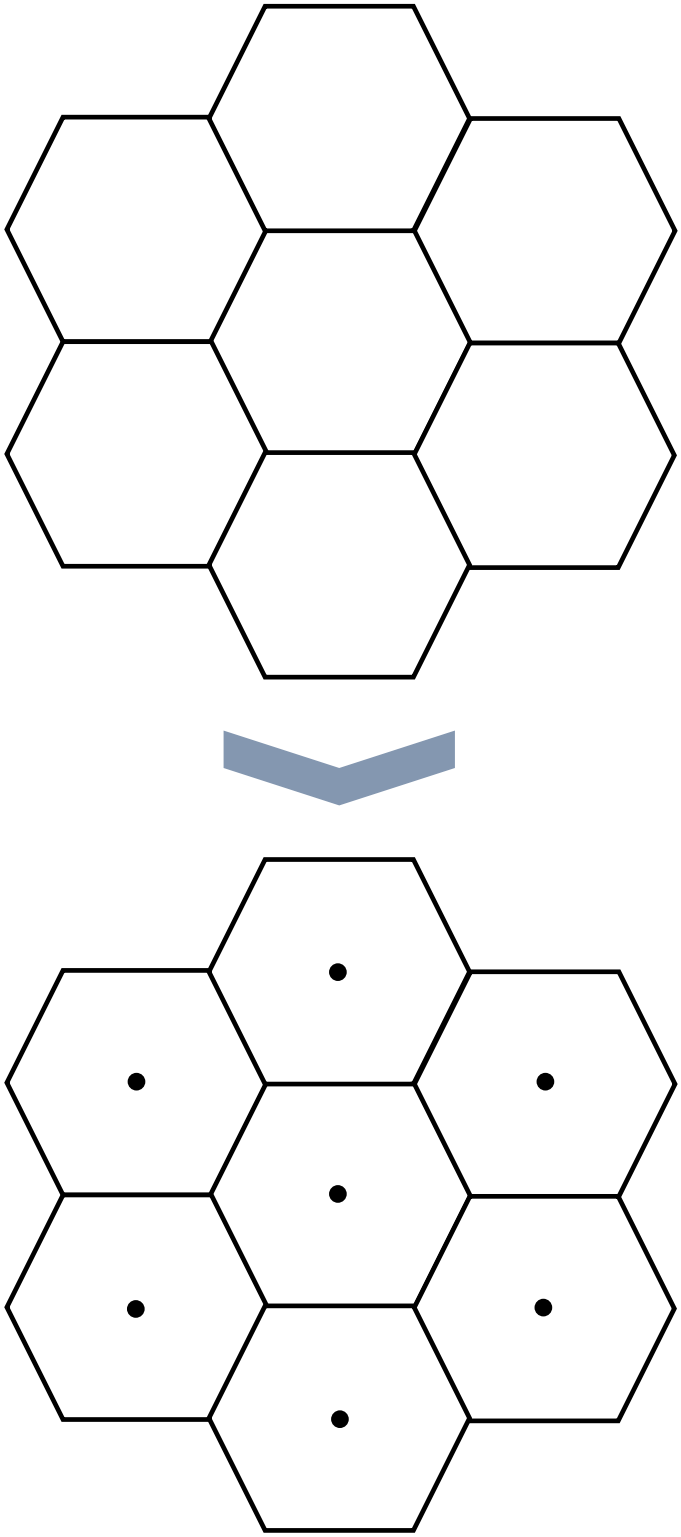
- Reallocate only part of the police department's
- Compact and contiguous territories to improve patrol
- Prevention of isolated departments

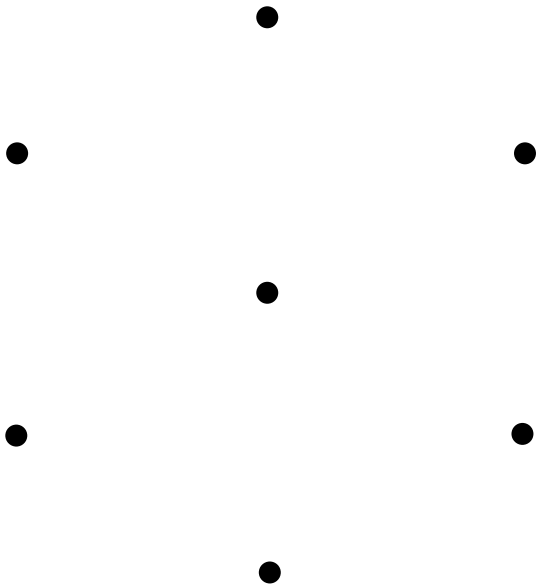
Basic Structure

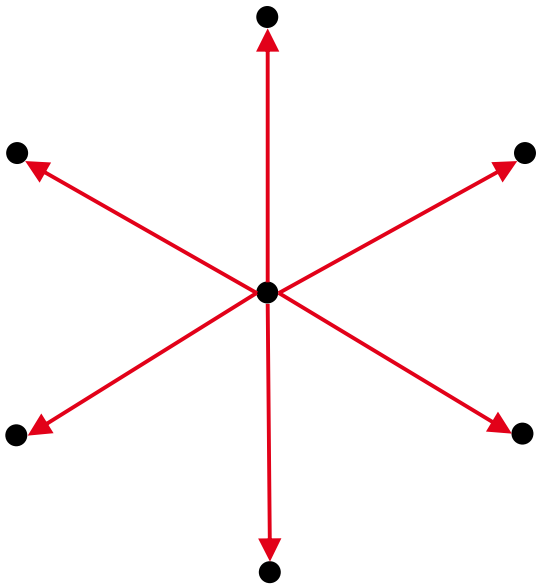


Question: How can we structure this?

- Model as a digraph with vertices and edges
- Each BA centroid becomes a vertex
- $\mathcal{J}$: set of BAs, indexed by j
- $\mathcal{I}$: set of potential district centres $(\mathcal{I} \subseteq \mathcal{J})$

Why Hexagons?





Question: Advantages of hexagons?

- Equal distances to all neighboring centroids
- Reduces sampling bias from edge effects [9]
- Special properties that help with the enforcement of compactness
- Better representation of urban geography

Response Time Components


Call Length
Dispatch Time
Driving Time

Question: How can we model response time?

- Call length is independent of territory
- Dispatch time is difficult to model
- Driving time can be minimized directly

> ⚠ Conclusion
>
> We focus on minimizing expected driving times between departments and incident locations.

Model Formulation

Let's build our model

step by step!

Key Model Components

Question: What could be our key model components?

. . .

- Basic areas (BAs) and potential department locations
- Driving times between basic areas
- Forecasted incident data
- Assignment decisions

. . .

Question: Which are sets, parameters, and variables?

Sets and Indices

- $\mathcal{J}$ - Set of BAs, indexed by j
- $\mathcal{I}$ - Set of potential district centres ($\mathcal{I} \subseteq \mathcal{J}$), indexed by i

. . .

> ℹ Note
>
> The depot locations are a subset of the basic areas!

Parameters

Question: What parameters do we need?

- p - Number of district centres (departments)
- $t_{i,j}$ - Expected driving times between i and j

...

> Tip
>
> Parameters should be carefully calibrated with real-world data!

Decision Variable(s)?

> We have the following sets:
>
> - BAs, indexed by $j \in \mathcal{J}$
> - Potential department locations, indexed by $i \in \mathcal{I}$

...

> Our objective is to:
>
> Minimize the expected response time of the emergency services by optimizing the assignment of BAs to departments.

...

Question: What decisions do we need to model?

Decision variable/s

- $X_{i,j}$: 1 if BA j assigned to department i, 0 otherwise

...

Question: What is the domain of our decision variable?

...

- $X_{i,j} \in \{0, 1\} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$

Let's build our

objective function!

Objective Function?

> Our objective is to:
>
> Minimize the expected response time of the emergency services by optimizing the assignment of BAs to departments.

...

Question: How do we minimize response time?

- We want to minimize total driving time
- Consider frequency of incidents in each BA
- Don't include fixed costs (handled by constraints)

Objective Function

Question: What could be our objective function?

...

$$\text{minimize} \quad \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} t_{i,j} \times X_{i,j}$$

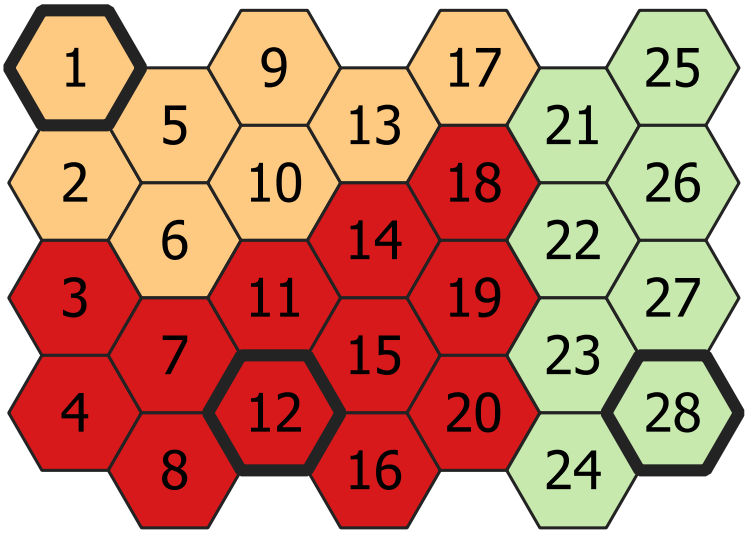
...

! Expected Driving Time

- Total driving time across all assignments
- Weighted by incident frequency
- Considers all possible BA-department pairs

Constraints

Key Constraints



Question: Constraints needed?

1. BA must have one department
2. Limit number of departments

3. Only assign active departments
4. Ensure contiguous districts
5. Maintain district compactness

Single Assignment Constraint?

Question: Why do we need this constraint?

. . .

- Each BA must be assigned to exactly one department
- Prevents overlapping jurisdictions
- Ensures complete coverage

. . .

> **i** We need the following variables:
>
> - $X_{i,j}$ - 1 if BA j assigned to department i, 0 otherwise

Single Assignment Constraint?

Question: What could the constraint look like?

. . .

$$\sum_{i \in \mathcal{I}} X_{i,j} = 1 \quad \forall j \in \mathcal{J}$$

. . .

> **i** Note
>
> Each BA must be assigned to exactly one department.

Department Count Constraint?

> **!** The goal of these constraints is to:
>
> Ensure that exactly p departments are opened.

. . .

i We need the following sets and variables:

- $\mathcal{I}$ - Set of potential department locations, indexed by i
- $\mathcal{J}$ - Set of BAs, indexed by j
- $X_{i,j}$ - 1, if BA j assigned to department i, 0 otherwise
- p - Number of departments

Department Count Constraint

Question: What could the constraint look like?

...

$$\sum_{i \in \mathcal{I}} X_{i,i} = p$$

...

Question: What happens if we have more departments than potential locations?

...

- We can't open more departments than there are locations
- The model will be infeasible

Active Department Constraint?

! The goal of these constraints is to:

Ensure that each BA is assigned to an active department, e.g. a department that is opened and that could dispatch vehicles.

...

i We need the following sets and variables:

- $X_{i,j}$ - 1, if BA j assigned to department i, 0 otherwise

Question: How do we ensure assignments only to active departments?

Active Department Constraint

$$X_{i,j} \leq X_{i,i} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$$

...

> **Note**
>
> This constraint creates a logical connection between department locations and BA assignments where BAs can only be assigned to opened departments.

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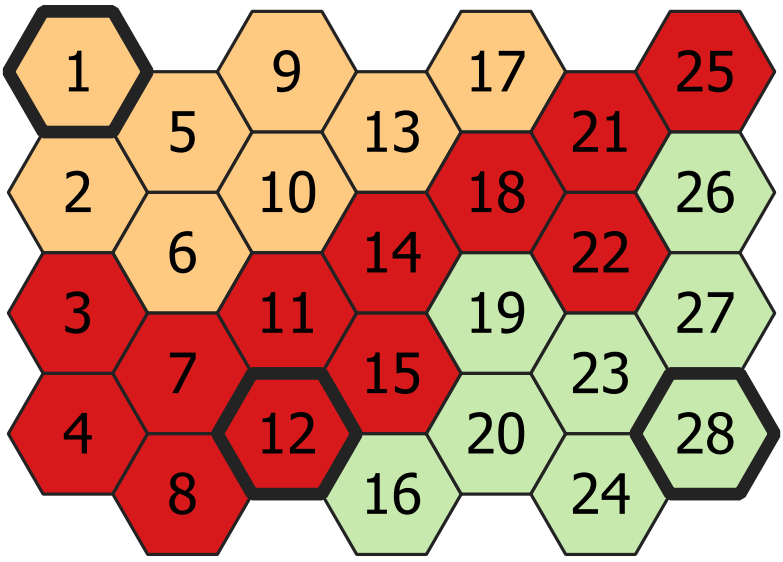
p-Median Problem

Contiguity and Compactness

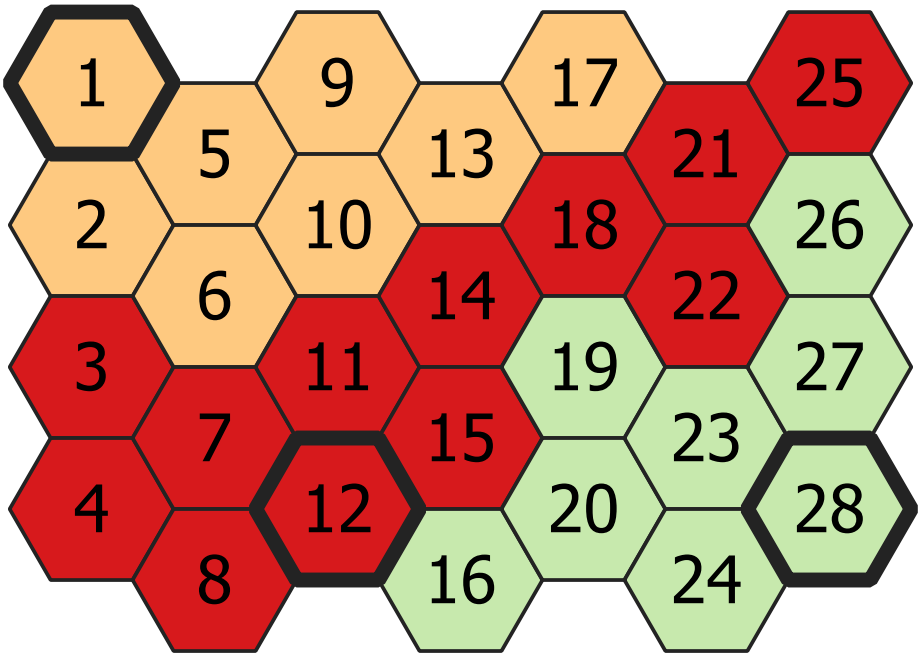
Contiguity Introduction



Question: Why is contiguity important?

- Prevents isolated areas
- Ensures contiguous patrol routes
- Maintains operational coherence

What is compactness?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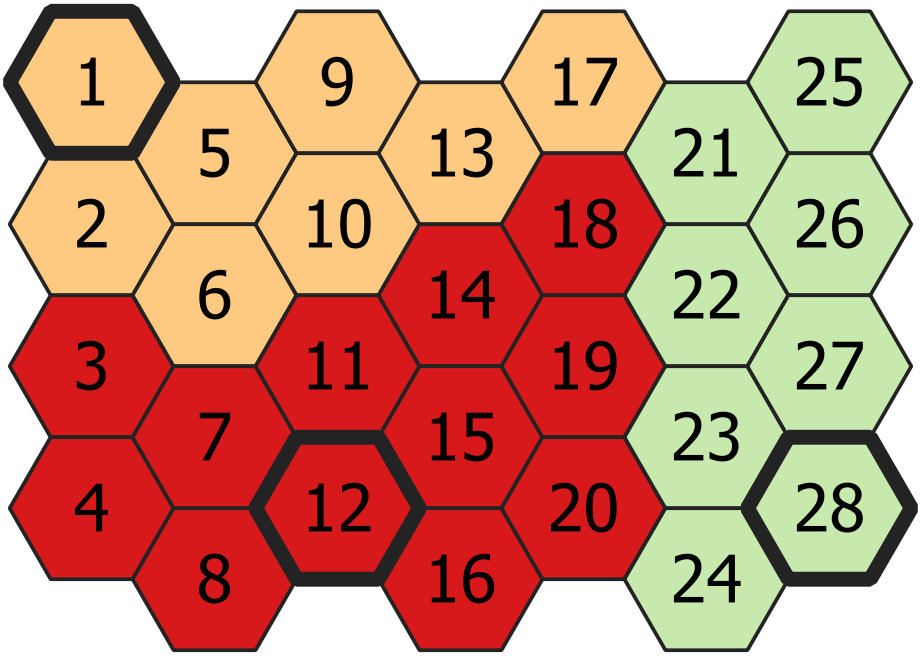
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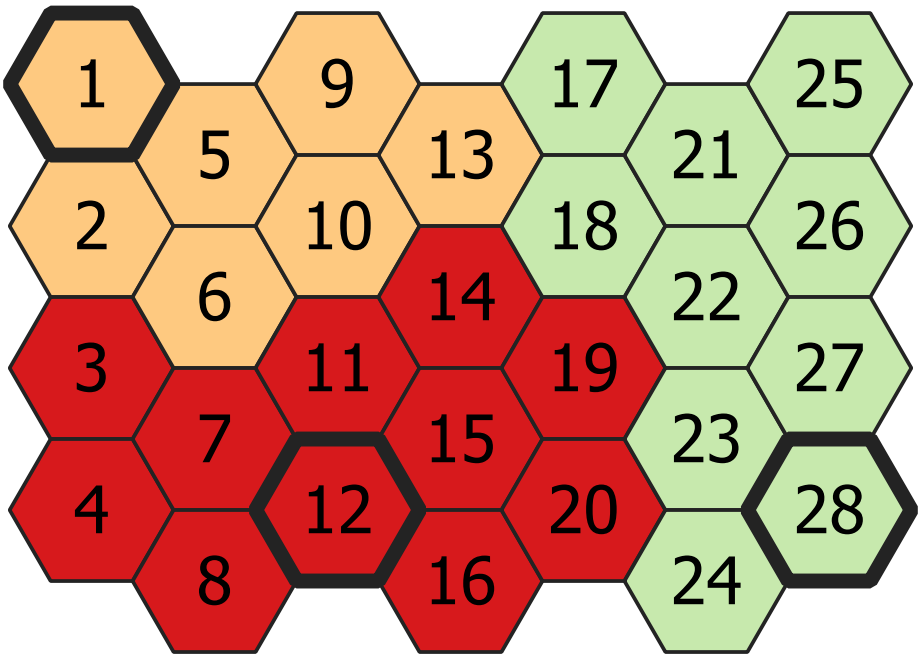

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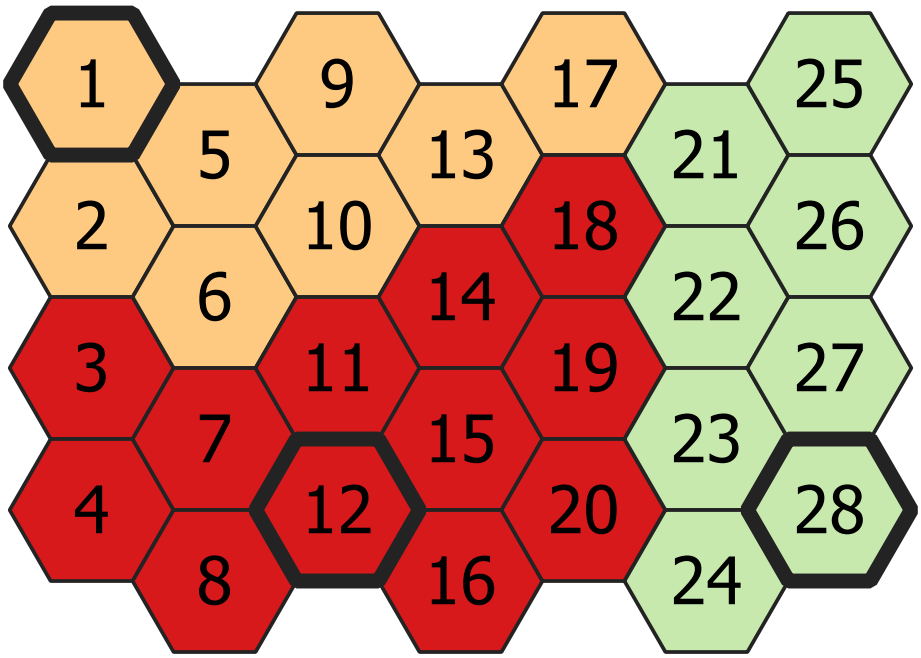
> **Compactness**
>
> Compactness has no univocal definition; a district is commonly declared compact if it is 'somehow round-shaped and undistorted' [10].

Contiguity and Compactness



Question: Are our resulting districts based on the model contiguous and compact?

- This depends on $t_{i,j}$
- If Euclidean distance
  - Districts will be contiguous
  - Likely of compact shape

Compactness p-Median

Question: Is this likely for police service districting?

- No, as we minimize the driving time within a city
- Highways, Tunnels, etc.
- Multiplied by the differing number of requested cars
- This can contribute to distorted district shapes

Contiguity Sets

Additional Set and Parameter

- $e_{i,j}$ - Euclidean distance between centroids
- $\mathcal{A}_j$ - Sets of BAs adjacent to BA j

...

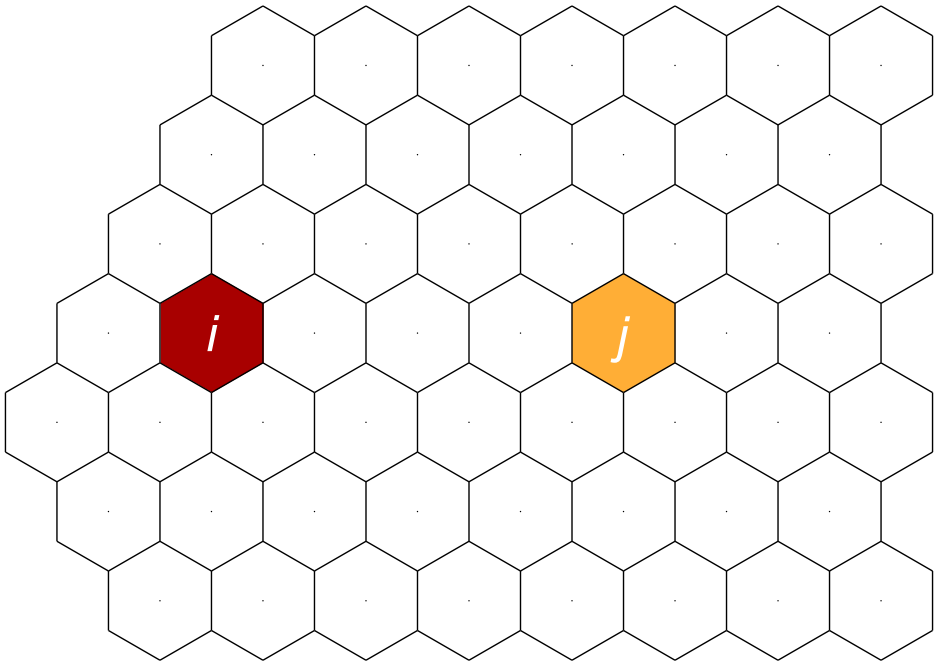
$$\mathcal{N}_{i,j} = \{v \in \mathcal{A}_j \mid e_{i,v} < e_{i,j}\} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$$

...

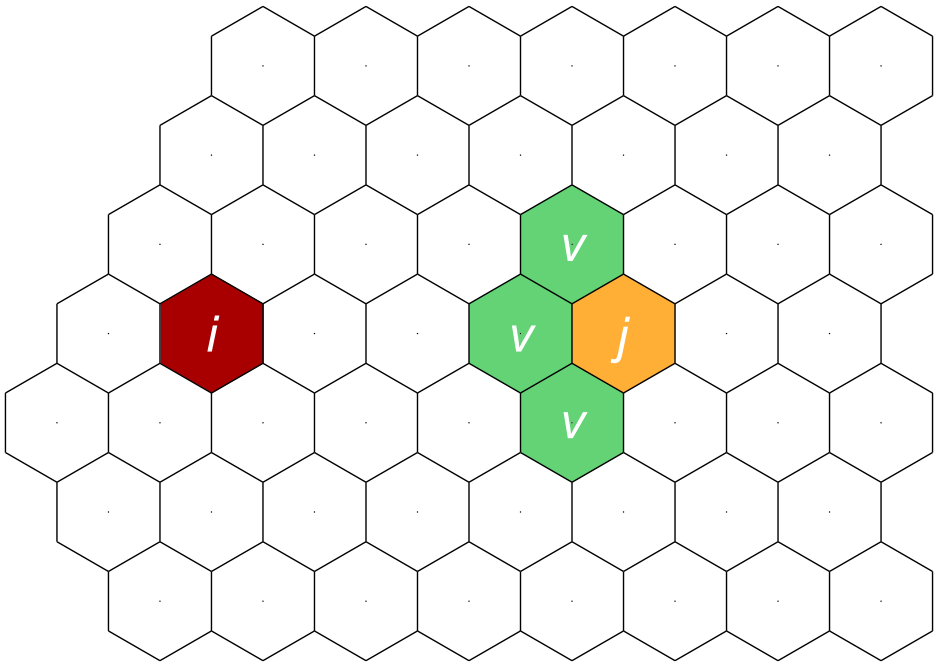
i The idea

BAs closer to department i than BA j on euclidian distance and adjacent to j!

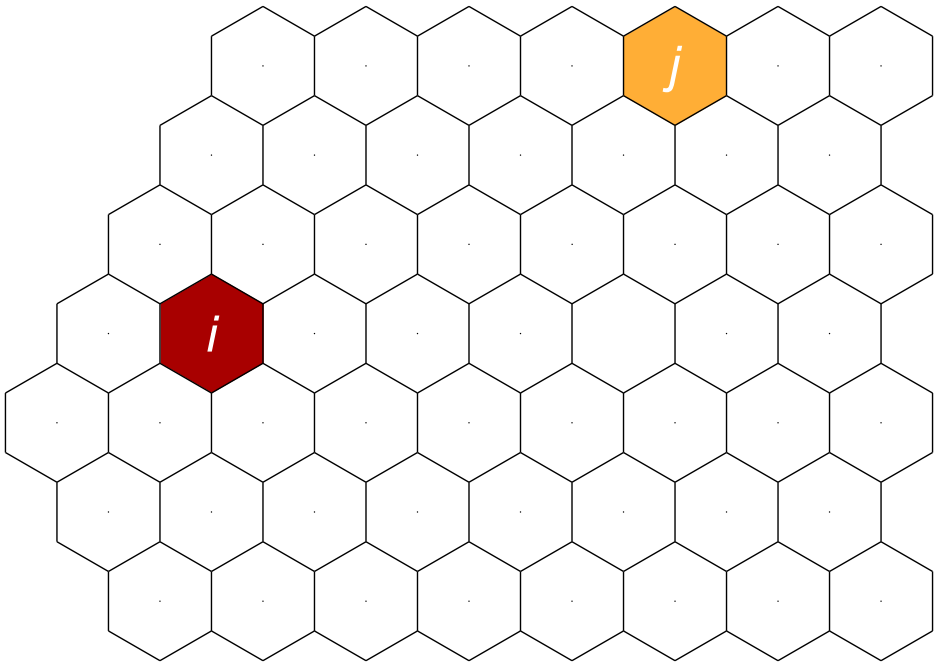
Example A



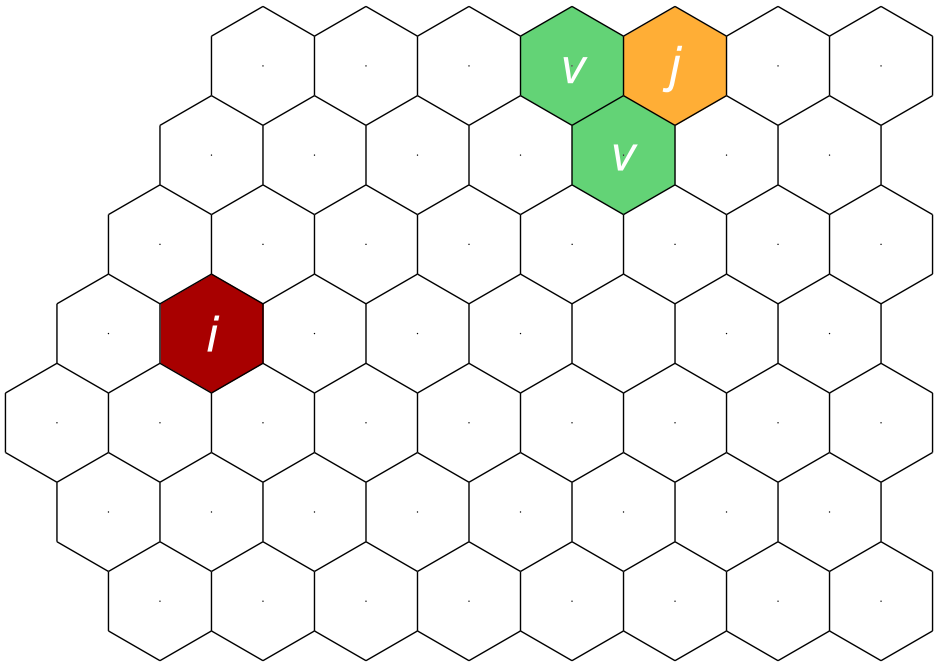
Example A



Example B



Example B



Enforcing Contiguity

All districts have to be contiguous

$$X_{i,j} \leq \sum_{v \in \mathcal{N}_{i,j}} X_{i,v} \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J} \setminus \mathcal{A}_i : i \neq j$$

...

> ! The idea
>
> At least one department has to be assigned to a BA that is adjacent to BA j and closer to department i!

Contiguity and Compactness

All districts have to be contiguous and compact

...

! The idea

At least one department has to be assigned to two BAs that are adjacent to BA j and closer to department i!

Comparison

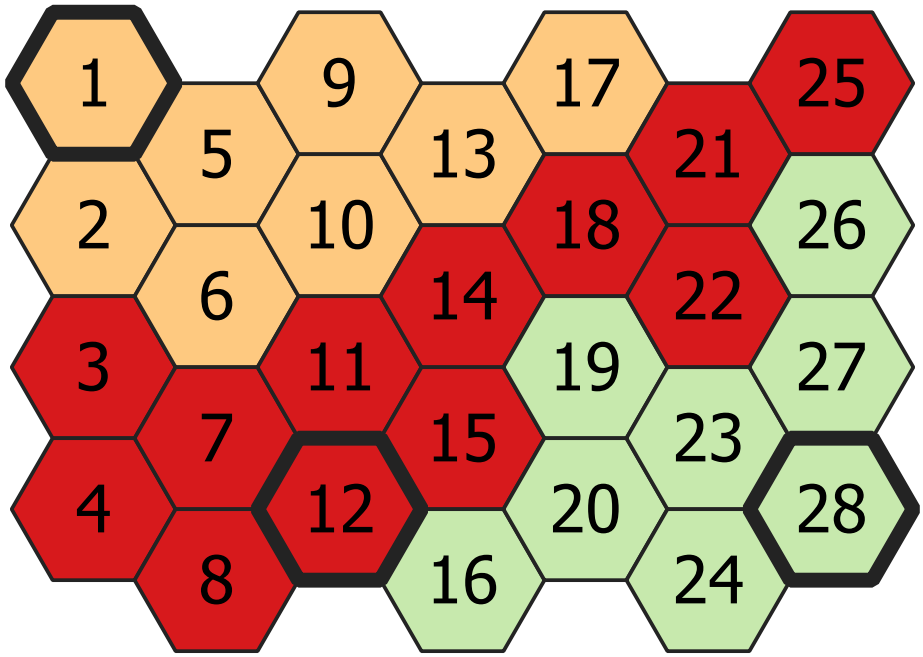


Figure 1: One department

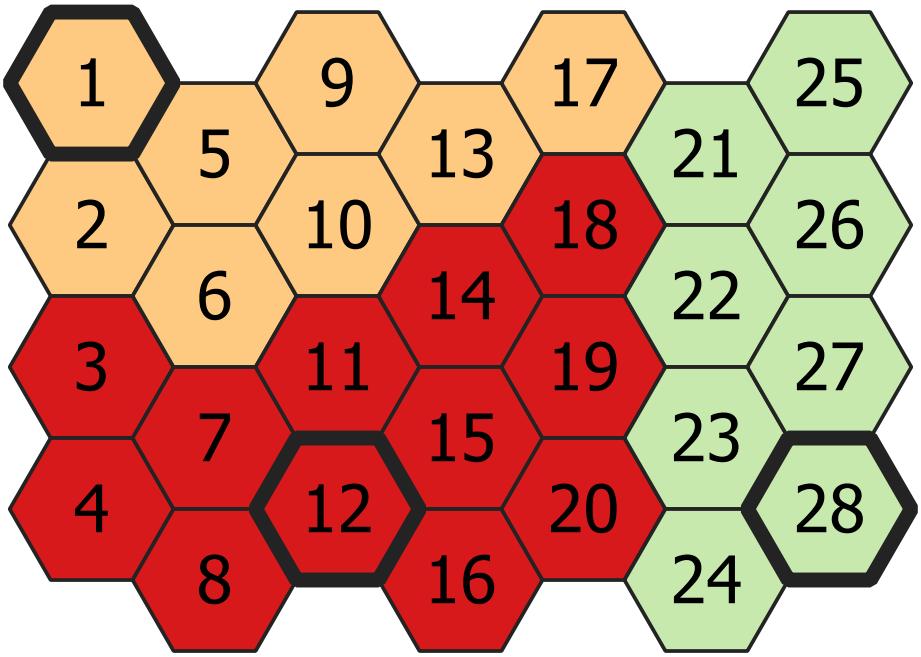


Figure 2: Two departments

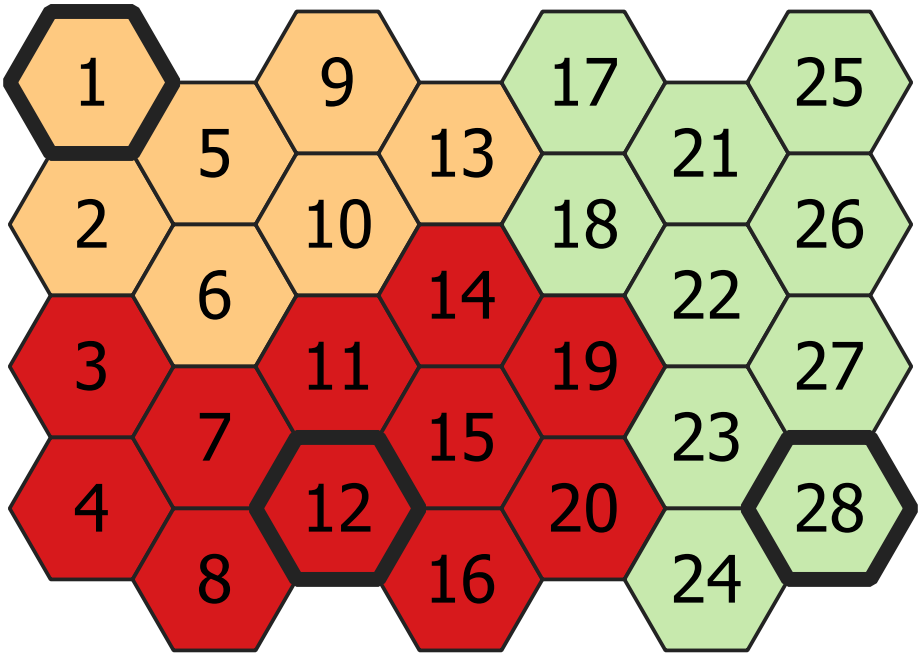


Figure 3: Up to three departments

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> **i** Why does this work?
>
> Due to the constraints, there is always a path back to the department if a BA is assigned to a department!

Model Characteristics

Characteristics

Questions: On model characteristics

- Is the model formulation linear/ non-linear?
- What kind of variable domains do we have?
- What do you think, can the model be solved quickly?
- Have we prevented isolated districts?

Model Assumptions

Questions: On model assumptions

- What assumptions have we made?
- Use Euclidean distances to approximate driving time?
- Can we rely on incident data collected by the police?

Implementation and Impact

Overview of Studies

Question: Where did we apply our model?

- Two distinct environments:
  1. Large metropolitan area (Germany)
  2. Rural region (Belgium)
- Different challenges and requirements
- Focus on response time optimization

German Metropolitan Case

. . .

- 1.8 mio incidents (2015-2019)
- ~20 department locations
- 1,596 basic areas
- Dense urban environment

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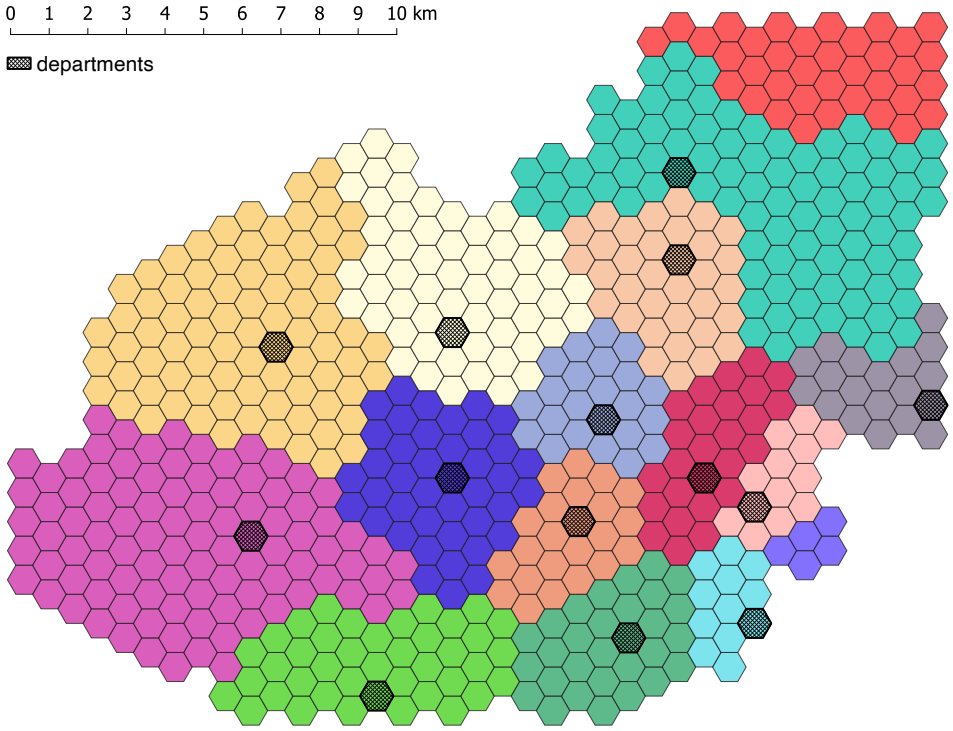
Goal: Redesign districts to improve response time.

German Metropolitan Results



Belgian Rural Case

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- 50,000 incidents (2019-2020)
- 2 existing + 1 planned location
- 1,233 basic areas
- Dispersed rural setting

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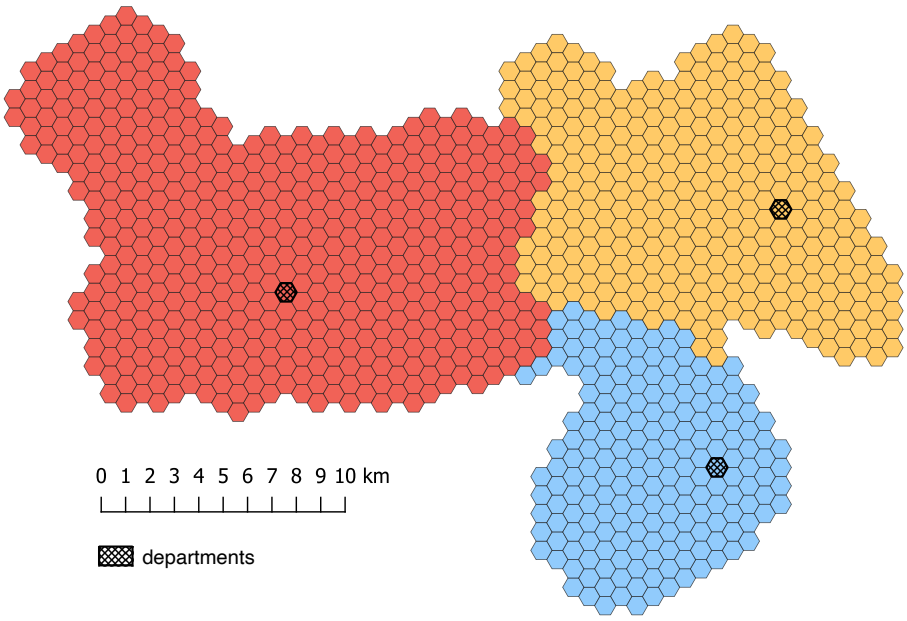
Goal: Optimize coverage with limited resources.

Belgian Rural Results



Simulation Framework

Question: How did we validate the results?

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- Spatial and temporal patterns
- Shift schedules
- Priority handling
- Rush hours
- Inter-district support
- Variable driving times

Results

- Response time reduction up to 14.52%
- Better workload distribution
- Improved coverage equity
- More efficient resource utilization

. . .

> ! Important
>
> All improvements are without additional staff!

Conclusions

1. Model adaptability crucial
2. Local context matters

3. Stakeholder buy-in essential
4. Data quality critical

...

> **Tip**
>
> Success requires balancing theoretical optimization with practical constraints!

Future Applications

Question: Where else could this approach be useful?

- Other emergency services
- Different urban contexts
- Resource allocation problems
- Service territory design

...

> **Note**
>
> The methodology is adaptable to various public service optimization scenarios.

Wrap Up

> **And that's it for todays lecture!**
>
> We now have covered districting problems and are ready to start solving some tasks in the upcoming tutorial.

Questions?

Literature

Literature I

For more interesting literature to learn more about Julia, take a look at the literature list of this course.

Literature II

Bibliography

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