

Lecture V - Planning in Breweries

Applied Optimization with Julia

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Introduction

Case Study



- Large brewery
- Brews and sells beverages
- Production planning by hand
- Planner has a **lot of experience**
- **But** will retire soon

Challenges



- Strong competition
- Customer demand is changing
- Craft beer gains popularity
- Variety of drinks is increasing
- Batch sizes are getting smaller

Different costs



- Plant can fill **multiple types**
- Time depends on **type and batch**
- **Changing type** leads to setup costs for preparation and cleaning
- **Unsold beer bottles** can be **stored in a warehouse**
- This leads to **inventory costs**

Learning Objectives

After this lecture, you will be able to:

- **Formulate** the Capacitated Lot-Sizing Problem (CLSP)
- **Explain** the trade-off between setup and inventory holding costs
- **Recognize and construct** Big-M constraints
- **Discuss** the assumptions and limits of a finite planning horizon

One Bottling Line

Question: Many beers share it. Where is the challenge?

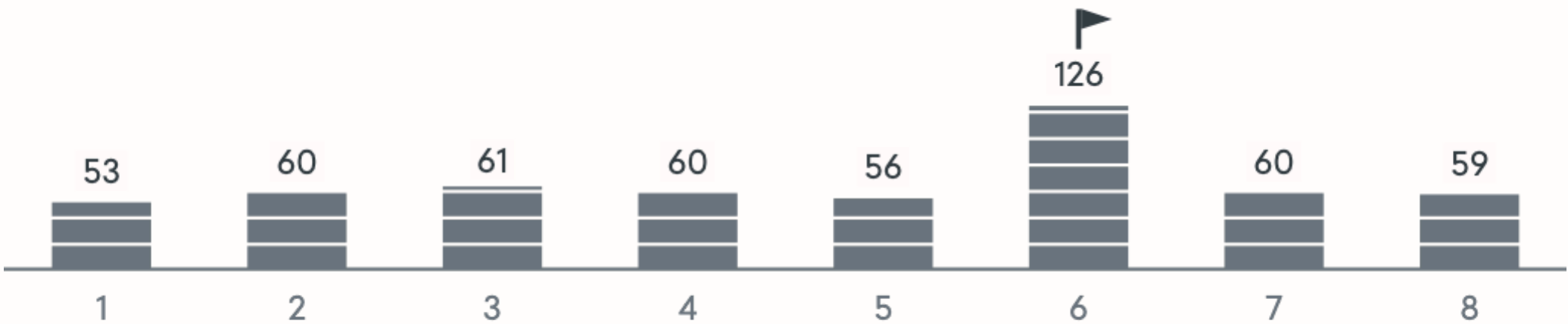


Photo: [Waldemar Brandt](#), Unsplash

Your Turn

The planner retires, and you take over. Tap a week to switch its bottling run on or off: a run bottles every crate until the next run. Each run costs 400 € for the setup, each crate costs 2 € per week in the warehouse. A flag marks the harbour festival.

Orders in crates



Bottling runs, tap a week



In the warehouse



Runs: 8 × 400 €

Stock: 0 × 2 €



You 3,200 €

Ready: press
Optimize to compare
with the best plan.

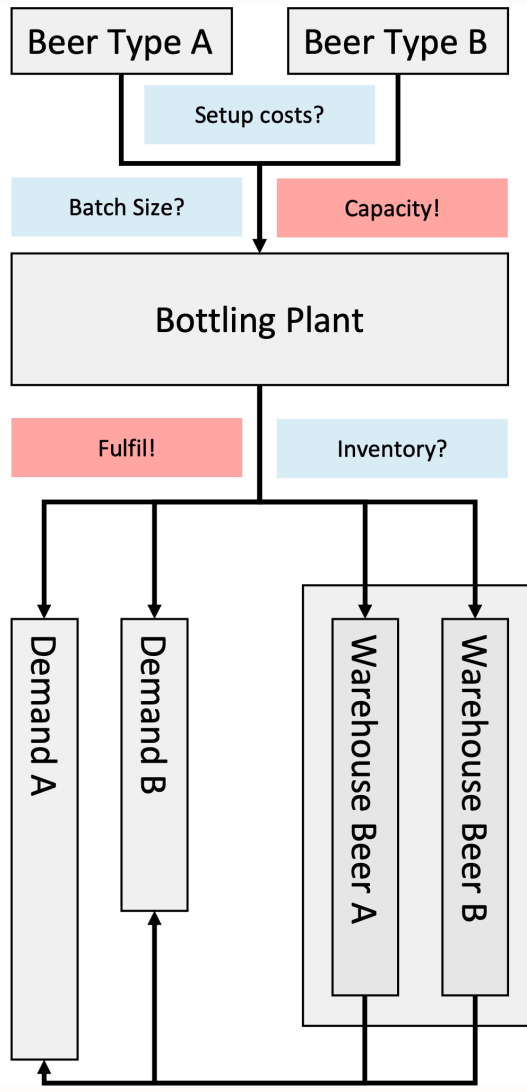
Optimize

Reset

New

Problem Structure

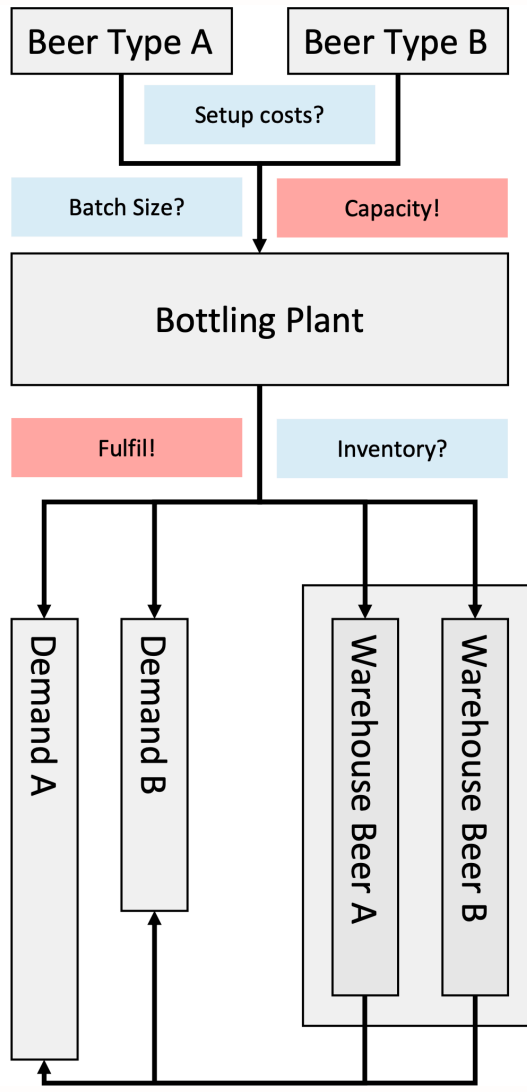
Objective



Question: What could be the objective?

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

Trade-Off



Question: What is the trade-off?

Larger batches require **less setup cost per bottle**, but increase the storage cost.

Available Sets

Question: What are sets again?

Sets are **collections of objects**.

Question: What could be the sets here?

- $\mathcal{I}$ - Set of beer types indexed by $i \in \{1, 2, \dots, |\mathcal{I}|\}$
- $\mathcal{T}$ - Set of time periods indexed by $t \in \{1, 2, \dots, |\mathcal{T}|\}$

Available Parameters

Question: What are possible parameters?

- a_t - Available time on the bottling plant in period $t \in \mathcal{T}$
- b_i - Time used for bottling one unit of beer type $i \in \mathcal{I}$
- g_i - Setup time for beer type $i \in \mathcal{I}$
- f_i - Setup cost of beer type $i \in \mathcal{I}$
- c_i - Inventory holding cost for one unit of beer type $i \in \mathcal{I}$
- $d_{i,t}$ - Demand of beer type $i \in \mathcal{I}$ in period $t \in \mathcal{T}$

Decision Variables?

i We have the following sets:

- Beer types indexed by $i \in \{1, 2, \dots, |\mathcal{I}|\}$
- Time periods of the planning horizon indexed by $t \in \{1, 2, \dots, |\mathcal{T}|\}$

! Our objective is to:

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

Question: What could be our decision variable/s?

Decision Variables

- $W_{i,t}$ - Inventory of type $i \in \mathcal{I}$ at the end of $t \in \mathcal{T}$
- $Y_{i,t}$ - 1, if type $i \in \mathcal{I}$ is bottled in $t \in \mathcal{T}$, 0 otherwise
- $X_{i,t}$ - Batch size of type $i \in \mathcal{I}$ in $t \in \mathcal{T}$

Model Formulation

Objective Function?

! Our objective is to:

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

Question: What could be our objective function?

i We need the following variables:

- $W_{i,t}$ - Inventory of type $i \in \mathcal{I}$ at the end of $t \in \mathcal{T}$
- $Y_{i,t}$ - 1, if type $i \in \mathcal{I}$ is bottled in $t \in \mathcal{T}$, 0 otherwise

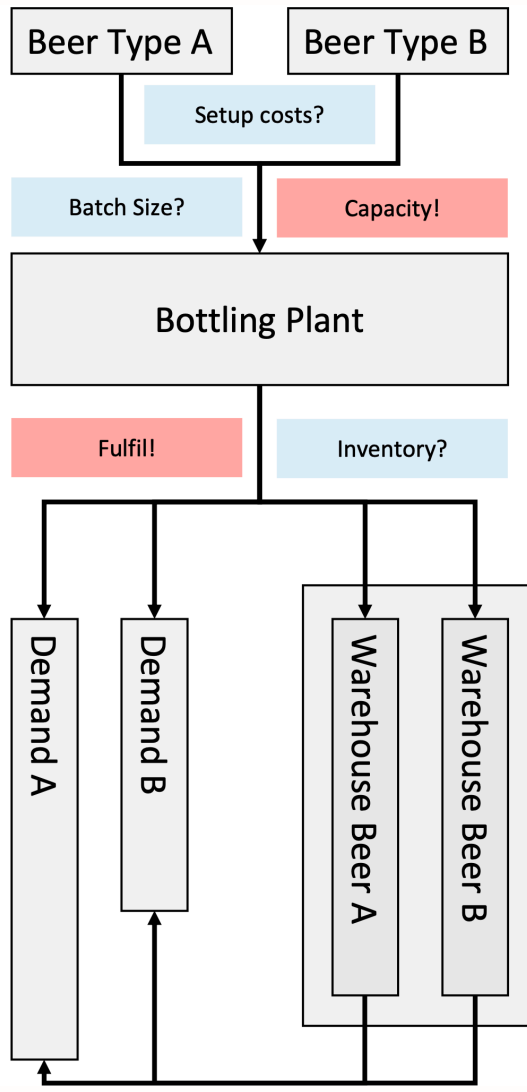
Objective Function

i We need the following parameters:

- f_i - Setup cost of beer type $i \in \mathcal{I}$
- c_i - Inventory holding cost for one unit of beer type $i \in \mathcal{I}$

$$\text{Minimize} \quad \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} (c_i \times W_{i,t} + f_i \times Y_{i,t})$$

Constraints



Question: What constraints?

- Transfer **unused** inventory
- **Fulfill** the customer demand
- **Set up** beer types
- Calculate **batch size** per set-up
- Compute **remaining** inventory
- **Limit** the bottling plant

Demand/Inventory Constraints?

! The goal of these constraints is to:

Consider the current inventory and batch sizes and compute the remaining inventory.

i We need the following variables and parameters:

- $W_{i,t}$ - Inventory of beer type $i \in \mathcal{I}$ at the end of period $t \in \mathcal{T}$
- $X_{i,t}$ - Batch size of beer type $i \in \mathcal{I}$ in $t \in \mathcal{T}$
- $d_{i,t}$ - Demand of beer type $i \in \mathcal{I}$ in period $t \in \mathcal{T}$

Question: What could the constraint look like?

Demand/Inventory Constraints

$$W_{i,t-1} + X_{i,t} - W_{i,t} = d_{i,t} \quad \forall i \in \mathcal{I}, t \in \mathcal{T}$$

with $W_{i,0} = 0$, as the warehouse starts empty.

i Remember, these are the variables and parameters:

- $W_{i,t}$ - Inventory of beer type $i \in \mathcal{I}$ at the end of period $t \in \mathcal{T}$
- $X_{i,t}$ - Batch size of beer type $i \in \mathcal{I}$ in $t \in \mathcal{T}$
- $d_{i,t}$ - Demand of beer type $i \in \mathcal{I}$ in period $t \in \mathcal{T}$

Why Fix the Starting Inventory?

Question: Why do we have to fix $W_{i,0} = 0$?

Otherwise, the solver could invent free starting inventory and would never need to bottle anything for the first weeks!

Setup Constraints?

! The goal of these constraints is to:

Set up beer types in all periods where the batch size is > 0 .

i We need the following variables and parameters:

- $Y_{i,t}$ - 1, if beer type $i \in \mathcal{I}$ is bottled in period $t \in \mathcal{T}$, 0 otherwise
- $X_{i,t}$ - Batch size of beer type $i \in \mathcal{I}$ in $t \in \mathcal{T}$
- $d_{i,t}$ - Demand of beer type $i \in \mathcal{I}$ in period $t \in \mathcal{T}$

Question: What could the second constraint be?

Setup Constraints

$$X_{i,t} \leq Y_{i,t} \times \sum_{\tau \in \mathcal{T}} d_{i,\tau} \quad \forall i \in \mathcal{I}, t \in \mathcal{T}$$

Question: Do you know this type of constraint?

This type of constraint is called a “**Big-M**” constraint!

- **M** (here $\sum_{\tau \in \mathcal{T}} d_{i,\tau}$) is a large number
- It is coupled with a binary variable (here $Y_{i,t}$)
- **Like an if-then constraint**

Capacity Constraints?

! The goal of these constraints is to:
Limit the capacity of the bottling plant per period.

i We need the following variables and parameters:

- $Y_{i,t}$ - 1, if beer type $i \in \mathcal{I}$ is bottled in period $t \in \mathcal{T}$, 0 otherwise
- $X_{i,t}$ - Batch size of beer type $i \in \mathcal{I}$ in $t \in \mathcal{T}$
- a_t - Available time on the bottling plant in period $t \in \mathcal{T}$
- b_i - Time used for bottling one unit of beer type $i \in \mathcal{I}$
- g_i - Setup time for beer type $i \in \mathcal{I}$

Capacity Constraints

Question: What could the third constraint be?

It uses more parameters than the previous constraints, but it is easier to understand.

$$\sum_{i \in \mathcal{I}} (b_i \times X_{i,t} + g_i \times Y_{i,t}) \leq a_t \quad \forall t \in \mathcal{T}$$

And that's basically it!

CLSP: Objective Function

The complete model is known as the **Capacitated Lot-Sizing Problem (CLSP)**.

$$\text{Minimize} \quad \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} (c_i \times W_{i,t} + f_i \times Y_{i,t})$$

! The goal of the objective function is to:

Minimize the combined setup and inventory holding cost while satisfying the demand and adhering to the production capacity.

CLSP: Constraints

$$W_{i,t-1} + X_{i,t} - W_{i,t} = d_{i,t} \quad \forall i \in \mathcal{I}, t \in \mathcal{T} \quad (W_{i,0} = 0)$$

$$X_{i,t} \leq Y_{i,t} \times \sum_{\tau \in \mathcal{T}} d_{i,\tau} \quad \forall i \in \mathcal{I}, t \in \mathcal{T}$$

$$\sum_{i \in \mathcal{I}} (b_i \times X_{i,t} + g_i \times Y_{i,t}) \leq a_t \quad \forall t \in \mathcal{T}$$

! Our constraints ensure:

Demand is met, inventory transferred, setup taken care of, and capacity respected.

CLSP: Variable Domains

$$Y_{i,t} \in \{0, 1\} \quad \forall i \in \mathcal{I}, t \in \mathcal{T}$$

$$W_{i,t}, X_{i,t} \geq 0 \quad \forall i \in \mathcal{I}, t \in \mathcal{T}$$

! The variable domains make sure that:

The binary setup variable is either 0 or 1 and that the inventory and batch size are non-negative.

Question: Why is a continuous batch size acceptable here?

Rounding a batch of thousands barely changes the cost.

Julia Implementation

Variables in Julia

Question: How could we formulate the variables in Julia?

```
1 using JuMP, HiGHS
2 beer_types = ["IPA", "Lager", "Stout"] # Beer types
3 periods = 1:3 # Time periods (weeks)
4 clsp_model = Model(HiGHS.Optimizer)
5 set_silent(clsp_model) # Hide the solver output
```

```
1 @variable(clsp_model, Y[i in beer_types, t in periods], Bin)
2 @variable(clsp_model, X[i in beer_types, t in periods] >= 0)
3 @variable(clsp_model, W[i in beer_types, t in 0:last(periods)] >= 0)
```

2-dimensional DenseAxisArray{JuMP.VariableRef,2,...} with index sets:
Dimension 1, ["IPA", "Lager", "Stout"]
Dimension 2, 0:3

And data, a 3×4 Matrix{JuMP.VariableRef}:

W[IPA,0]	W[IPA,1]	W[IPA,2]	W[IPA,3]
W[Lager,0]	W[Lager,1]	W[Lager,2]	W[Lager,3]
W[Stout,0]	W[Stout,1]	W[Stout,2]	W[Stout,3]

Note, how we declare the inventory W also for period 0!

Objective Function in Julia

$$\text{Minimize } \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} (c_i \times W_{i,t} + f_i \times Y_{i,t})$$

```
1 c = Dict{"IPA" => 0.1, "Lager" => 0.1, "Stout" => 0.1} # Holding costs
2 f = Dict{"IPA" => 5000, "Lager" => 4000, "Stout" => 6000} # Setup costs
```

```
1 @objective(clsp_model, Min,
2     sum(c[i] * W[i,t] + f[i] * Y[i,t]
3         for i in beer_types, t in periods)
4 )
```

Data for our Small Brewery

We need the demand $d_{i,t}$, capacity a_t , and times b_i, g_i :

```
1 d = Dict( # Demand per beer type and week
2     "IPA"    => [300, 800, 500],
3     "Lager"  => [600, 500, 800],
4     "Stout"  => [200, 300, 250],
5 )
6 a = [1800, 1800, 1800] # Capacity per week in minutes
7 b = Dict("IPA" => 0.9, "Lager" => 0.6, "Stout" => 1.2) # Min. per bottle
8 g = Dict("IPA" => 60, "Lager" => 40, "Stout" => 80) # Setup in minutes
```

Constraints in Julia

The constraints map almost one-to-one from the math:

```
1 @constraint(clsp_model, [i in beer_types],  
2     W[i,0] == 0) # The warehouse starts empty  
3 @constraint(clsp_model, [i in beer_types, t in periods],  
4     W[i,t-1] + X[i,t] - W[i,t] == d[i][t])
```

```
1 @constraint(clsp_model, [i in beer_types, t in periods],  
2     X[i,t] <= sum(d[i]) * Y[i,t]) # Big-M setup constraint  
3 @constraint(clsp_model, [t in periods],  
4     sum(b[i] * X[i,t] + g[i] * Y[i,t] for i in beer_types) <= a[t])
```

Solving the Model

```
1 optimize!(clsp_model)
2 println("Status: ", termination_status(clsp_model))
3 println("Total cost: ", objective_value(clsp_model))
```

Status: OPTIMAL

Total cost: 28130.0

Of the 28,130 total cost, 28,000 are setup costs and only 130 are inventory holding costs — but what does the plan look like?

The Production Plan

```
1 for i in beer_types
2     println(rpad(i, 8),
3         "X = ", rpad(string([round(Int, value(X[i,t])) for t in periods]), 18),
4         "W = ", [round(Int, value(W[i,t])) for t in periods])
5 end
```

IPA	X = [300, 1300, 0]	W = [0, 500, 0]
Lager	X = [600, 500, 800]	W = [0, 0, 0]
Stout	X = [750, 0, 0]	W = [550, 250, 0]

- **Stout** is bottled **only once**: week 1 covers all three weeks, saving two setups at 6,000 each
- **Lager** is bottled every week, as the plant capacity is too tight to batch it
- **IPA** batches the demand of week 3 into week 2

Model Characteristics

Recap on some Basics

There exist several types of optimization problems:

- **Linear (LP):** Linear constraints and objective function
- **Mixed-integer (MIP):** Linear constraints and objective function, but discrete variable domains
- **Quadratic (QP):** Quadratic constraints and/or objective
- **Non-linear (NLP):** Non-linear constraints and/or objective
- **And more!**

Recap on Solution Algorithms

- **Simplex algorithm** to solve LPs
- **Branch & Bound** to solve MIPs
- **Outer-Approximation** for mixed-integer NLPs
- **Matheuristics** (e.g., Fix-and-Optimize, ...)
- **Metaheuristics** (e.g., tabu search, genetic algorithms, ...)
- **Decomposition** methods (Lagrange, Benders, ...)
- **Heuristics** (greedy, construction method, n-opt, ...)
- **Graph theoretical methods** (network flow, shortest path)

Model Characteristics

Questions: On model characteristics

- Is the model formulation linear/ non-linear?
- What kind of variable domains do we have?
- What kind of solver could we use?
- Can the Big-M constraint be tightened?

Tightening the Big-M

Question: How small can we make M without cutting off solutions?

- Production in period t only serves demand from t onward:

$$M_{i,t} = \sum_{\tau \in \mathcal{T} | \tau \geq t} d_{i,\tau}$$

- Capacity limits a batch as well: $M_{i,t} = (a_t - g_i)/b_i$

The **smaller** of the two is the tighter bound — and tighter formulations are usually solved faster!

Model Assumptions

Questions: On model assumptions

- What assumptions have we made?
- What is the problem with the planning horizon?
- Any idea how to solve it?

End-of-horizon effect

The model plans no inventory for demand after week $|\mathcal{T}|$, as it does not know about it. In practice, this is solved by re-planning on a **rolling horizon**.

Impact

Can this be
applied?

Scale as a Problem

Solving the problem with commercial solvers is not tractable, as it takes too long.



Scale of the Case Study

- 220 finished products
- 100 semi-finished products
- 13 production resources
- 8 storage resources
- 3 main production levels
- 52 weeks planning horizon

Our CLSP is the **single-level core**: the real case is multi-level, with semi-finished products and storage.

Any idea what
could be done?

Heuristics and Optimization

- Multi-level Capacitated Lot-Sizing Problem
- Heuristic fix and optimize approach ¹
- Operating cost reduction by 5% and planning effort by 40%

 And that's it for today's lecture!

We now have covered the basics of the CLSP and are ready to start solving some tasks in the upcoming tutorial.

Questions?

Literature

Literature

For more interesting literature to learn more about Julia, take a look at the [literature list](#) of this course.

Mickein, Markus, Matthes Koch, and Knut Haase. 2022. “A Decision Support System for Brewery Production Planning at Feldschlösschen.” *INFORMS Journal on Applied Analytics* 52 (2): 158–72.